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HEIGHTS AND WEIGHTS OF 275 PUBLIC SCHOOL GIRLS FOR CONSECUTIVE AGES 7 TO 16 YEARS, INCLUSIVE

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The growth study conducted by Dr. W. F. Dearborn in which a large variety of measurements were taken annually on a large number of public school children in the twelve grades enables one to find 275 girls who were measured for height and weight each year from age 7 to age 16, inclusive.[2] The mean height in centimeters, the increments in height[3] and the standard deviation, the coefficient of variation, followed by corresponding items relative to weight are given in table 1.

TABLE 1

AGE	HEIGHT (CM.)	DIFF.	S. D.	C. V.	WEIGHT (KG.)	DIFF.	S. D.	C. V.
7	119.12		5.88	4.9%	22.70		3.32	14.6%
		5.45				2.54		
8	124.57		6.12	4.9%	25.24		4.22	16.7%
		5.34				2.73		
9	129.91		6.47	5.0%	27.97		4.93	17.6%
		5.43				3.28		
10	135.34		6.97	5.2%	31.25		6.06	19.4%
		6.11				4.29		
11	141.45		7.74	5.5%	35.54		7.51	21.1%
		6.60				5.07		
12	148.05		8.00	5.4%	40.61		8.65	21.3%
		5.35				5.23		
13	153.40		7.42	4.8%	45.84		9.24	20.1%
		3.30				4.03		
14	156.70		6.75	4.3%	49.87		9.04	18.1%
		1.76				2.73		
15	158.46		6.49	4.1%	52.60		9.11	17.3%
		0.82				1.24		
16	159.28		6.37	4.0%	53.84		8.93	16.6%

The correlation coefficients for height with height at different years of age are given in the northeast corner (in italic type), corresponding results for weights are given in the southwest corner (in ordinary type) and the

correlations of height with weight for each of the ten years is given in the main diagonal (in boldface type), of table 2.

TABLE 2

AGE	HEIGHTS 7	8	9	10	11	12	13	14	15	16
7	**0.767**	*0.987*	*0.980*	*0.957*	*0.920*	*0.897*	*0.887*	*0.866*	*0.836*	*0.810*
8	0.890	**0.725**	*0.989*	*0.969*	*0.934*	*0.914*	*0.904*	*0.882*	*0.850*	*0.824*
9	0.880	0.920	**0.721**	*0.986*	*0.954*	*0.927*	*0.909*	*0.881*	*0.844*	*0.814*
10	0.835	0.896	0.932	**0.715**	*0.979*	*0.947*	*0.911*	*0.865*	*0.816*	*0.780*
11	0.810	0.871	0.906	0.958	**0.711**	*0.974*	*0.923*	*0.855*	*0.790*	*0.747*
12	0.793	0.856	0.882	0.936	0.967	**0.707**	*0.964*	*0.887*	*0.810*	*0.763*
13	0.755	0.825	0.840	0.892	0.921	0.954	**0.607**	*0.961*	*0.901*	*0.860*
14	0.773	0.812	0.818	0.842	0.866	0.892	0.944	**0.521**	*0.974*	*0.948*
15	0.744	0.771	0.773	0.777	0.790	0.816	0.880	0.953	**0.430**	*0.989*
16	0.732	0.759	0.756	0.755	0.762	0.775	0.839	0.916	0.965	**0.424**
	WEIGHTS									

It will be noticed that the correlation of height with weight falls off from 0.77 at age 7 to 0.42 at age 16, showing how sharply the girls are differentiating in this aspect of their body build. The weight-weight correlations are in general less than the height-height correlations for corresponding years; the few exceptions may not be significant but the way they bunch together tempts one to place more emphasis on the matter than the slight differences in the coefficients would of themselves justify. The height-height correlation for consecutive years is lowest for the pair of years 13–14, the weight-weight correlation for consecutive years is lowest at the start, 7–8, but dips again slightly for the years 13–14. The correlation of height at 7 with height at 16 (which should be near the terminal height in girls) is $r = 0.81$, $r^2 = 0.66$, $1 - r^2 = 0.34$, which means that the variance in the height at 16 is 66% "controlled" by a knowledge of the height at age 7; but as for the ages 11 and 16 the values are $r = 0.75$, $r^2 = 0.56$, $1 - r^2 = 0.44$, the variance in height at 16 is only 56% controlled by knowledge of the height at age 11.

[1] The data used here were transferred to punched cards by Dr. Ann Stewart, Fellow of the General Education Board, to help in establishing norms against which to project a detailed case study of the girls who at some time were growing in a very exceptional manner. The calculations for tables 1 and 2 were made by Miss Constance Bennett.

[2] By girls 7 are meant girls between 7 and 8 years old. The group of 275 was divided into 6 subgroups, of variable numbers, of girls 7 yr. to 7 yr. 2 mo., etc., to see whether the means of the subgroups differed from the expected values by more than the error to be expected in sampling and they did not, albeit the number of girls in the six different two-months age groups was more variable than would have been expected by chance.

[3] The differences for successive years should be very much steadier statistically in a table constructed for the same girls in successive years than in one constructed for equal sized groups of different girls at the different ages, because of the high correlations between the figures for consecutive years, the lowest of which is $r = 0.96$ for height and $r = 0.89$ for weight.

TRANSPLANTATION OF FETAL TISSUES BETWEEN RABBITS AND RATS

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More or less isolated organ rudiments and portions of young rabbit embryos are capable of realizing their developmental potencies in such abnormal environments as those furnished by the omental bursa, kidney, etc., of the adult rabbit. The transplants are incorporated by host tissue and differentiation of the contained tissues and organ rudiments ensues. Information has been secured as to the reaction between host and donor tissues, the size of the transplant suitable for successful incorporation, the fate of the transplant and the survival time of the graft. Whole and portions of very young stages have been transplanted to determine something of their organization. Moreover, young stages will undergo some further growth when cultured *in vitro*.

With the above information as a control, fetal tissues of rat and rabbit embryos have been transplanted to the organs of adult rabbits and rats for the purpose of studying their capacity for differentiation in the more abnormal environment offered by a host of another species. It has seemed of interest to study the fate of embryonic tissue in a foreign adult host in the light of what is known about the fate of embryonic rat tissue on the chorioallantoic membrane of the developing chick,[2,3] and of adult mouse tissue transplanted to the rat and vice versa.[4] The material consisted of eye, ear, spinal cord, brain, somite, etc., rudiments and portions of 9–13-day old rat and rabbit embryos. Fetal tissue of the rabbit was introduced beneath the capsule of the rat kidney, while that of the rat was placed both in the kidney and in the omental bursa of the rabbit. Adults of both sexes were used as well as pregnant and nonpregnant females. The rat omentum is not suitable for this work. The grafts were recovered for fixation and study at daily intervals from the second to the tenth day after transplantation.

The following is a brief summary of the results secured thus far. Incorporation of the transplants proceeds in both hosts in very much the same fashion and at about the same rate. The per cent of successful grafts recovered is small. The grafts are always small, inconspicuous, white masses in both the kidney and the omentum, although young stages may be pink in color. Host after host failed to show any sign of a graft or at most only white thickened areas. The survival time for grafts is short and the foreign body is removed much more rapidly than is the case of grafts in hosts of the same species. There is practically no variation between rat and rabbit

grafts. This agrees closely with the observation of Loeb[4] even though in the present study embryonic tissue was used for transplantation instead of adult tissue.

Histologically the reaction of the host tissue toward the foreign body is quite similar to that previously described for omental grafts of rabbit embryonic tissue in hosts of the same species, only more intensified. Incorporation proceeds but degenerative changes are soon in evidence, and the grafts survive for a short time only. No difference could be detected in the quality and quantity of the grafts between males and females, or between pregnant and nonpregnant females as hosts. Many cases show no reaction on the part of the membrane or traces of the transplant, while others may show either (1) a slight thickening of tissue in the region where the transplant was originally inserted, (2) a mass of tissue containing degenerated embryonic tissue or (3) a larger graft mass containing a variable number of embryonic tissues which may show growth and differentiation beyond the stage at transplantation and which may have started to degenerate, or which have undergone very little differentiation. Even the best grafts are not without evidences of degeneration. Six to 8 days is about the longest survival of the most successful grafts. After this maximum period, degeneration proceeds at a rapid rate. The few grafts recovered on the tenth day reveal only small masses of white, fibrous tissue containing no embryonic tissue with the possible exception, in a very few cases, of bits of degenerated cartilage and epidermis.

Incorporation is of a shallow type with little invasion of host tissue into the substance of the transplant. In the case of rat tissue in the rabbit, the transplants seem to stand more chance for survival in the kidney than in the omentum, as more grafts were secured from the former. This may be accounted for by the fact that the omentum has been described as a storehouse of potential phagocytes. Clearly the phenomenon of tissue specificity must be initiated quite early in the course of development, and embryonic tissues, at least at the stages used in the present study, are very sensitive to the abnormal environment to which transplantation is made.

Portions of older embryos survive better than isolated rudiments and certain of the contained tissues or even organ rudiments undergo some further differentiation. Thus far no endodermal derivatives have been found in the grafts. Harder tissues such as cartilage and epidermis grow and differentiate while softer ones such as those of the eye and ear are malformed. Nervous tissue begins to degenerate early in graft life. Limb buds did not differentiate.

The most successful grafts which were secured show a much less abundant and poorer type of differentiation than that of the control, or that described by Nicholas and Rudnick[2] for transplants of rat embryonic tissue to the chorioallantoic membrane of the chick. While differentiation is irregular,

abnormal and retarded, yet these preliminary results do show that some rat and rabbit embryonic tissues are capable of undergoing growth and differentiation in these very foreign environments. But since the reaction of the host tissues against the transplant is initiated more quickly and progresses more rapidly than in a host of the same species, differentiation is hindered or inhibited before the embryonic tissues have much chance to express their potentialities.

One possible suggestion to account for the discrepancy between these results and those of Nicholas and Rudnick, is that transplantation was made to adult mature organs. Although in the present instance transplantation was made to a more closely related animal, yet the results are not at all comparable. The difference would seem to lie in the fact that embryonic tissue (i.e., chorioallantoic membrane) is more tolerant toward the presence of other embryonic tissues, even though they are from an embryo of an entirely different class, than is mature tissue toward embryonic tissue from a closer relationship. Loeb[4] concludes that the failure of heterotransplants of adult mammalian tissue is due to faulty vascularization and more especially to the toxic effect of the body fluid of the host. He does not think that it is primarily the destructive action of connective tissue cells, lymphocytes and polymorphonuclear leucocytes of the host, although in some cases these factors apparently help to destroy living structures of the graft.

[1] This study was carried out with the aid of a grant for research from the NATIONAL RESEARCH COUNCIL.

[2] Hiraiwa, Y. K., *Jour. Exp. Zoöl.*, **49**, 441 (1927); *Folia Anat. Japonica*, **8**, 157 (1930).

[3] Nicholas, J. S., and Rudnick, D., *Jour. Exp. Zoöl.*, **66**, 193 (1933); *Proc. Soc. Exp. Biol. Med.*, **29**, 325 (1931).

[4] Loeb, L., *Amer. Nat.*, **69**, 239 (1935).

THE TORSION OF A CIRCULAR CYLINDER

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Saint-Venant's solution of the torsion problem is based on the assumption that a transverse section of any uniform cylinder suffers no distortion in its own plane provided the cylinder is long and the section is not close to the ends. The assumption agrees well with experiment and simplifies the mathematical analysis. The problem of the circular cylinder is here solved without it in order to find what effects arise from the boundary conditions at the ends.

The double subscript symbol p_{mn} denotes a stress in the n direction on a plane normal to the m direction, where $m = 1, 2, 3$ and $n = 1, 2, 3$ indicate the first (r), second (θ) and third (z) directions of the cylindrical coördinates (r, θ, z). The strain e_{mn} has no strictly analogous meaning. The following single subscript notation applies to any set of orthogonal axes.

NORMAL STRESS	SHEAR STRESS	LINEAR STRAIN	SHEAR STRAIN
$p_1 = p_{11}$	$q_1 = p_{23} = p_{32}$	$e_1 = e_{11}$	$\gamma_1 = e_{23} = e_{32}$
$p_2 = p_{22}$	$q_2 = p_{31} = p_{13}$	$e_2 = e_{22}$	$\gamma_2 = e_{31} = e_{13}$
$p_3 = p_{33}$	$q_3 = p_{12} = p_{21}$	$e_3 = e_{33}$	$\gamma_3 = e_{12} = e_{21}$

When the stresses are independent of θ, the equations that define the equilibrium state of a weightless element $dr \cdot rd\theta \cdot dz$ are, the z axis being along the central axis of the shaft or cylinder,

$$\frac{\partial p_1}{\partial r} + \frac{\partial q_2}{\partial z} + \frac{p_1 - p_2}{r} = 0 \tag{1}$$

$$\frac{\partial q_3}{\partial r} + \frac{\partial q_1}{\partial z} + \frac{2q_3}{r} = 0 \tag{2}$$

$$\frac{\partial q_2}{\partial r} + \frac{\partial p_3}{\partial z} + \frac{q_2}{r} = 0 \tag{3}$$

If the displacements at any point (r, θ, z) are (ξ, η, ζ), and are independent of θ,

$$p_m = \frac{E}{1 + \sigma} e_m + \lambda \Delta, m = 1, 2, 3 \tag{4}$$

$$e_1 = \frac{\partial \xi}{\partial r}, e_2 = \frac{\xi}{r}, e_3 = \frac{\partial \zeta}{\partial z} \tag{5}$$

$$\Delta = e_1 + e_2 + e_3$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (r\xi) + \frac{\partial \zeta}{\partial z} \tag{6}$$

$$\frac{q_1}{G} = \gamma_1 = \frac{\partial \eta}{\partial z}, \frac{q_2}{G} = \gamma_2 = \frac{\partial \xi}{\partial z} + \frac{\partial \zeta}{\partial r}, \frac{q_3}{G} = \gamma_3 = \frac{\partial \eta}{\partial r} - \frac{\eta}{r} \tag{7}$$

$$\lambda = \frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)} = \frac{2G\sigma}{1 - 2\sigma},$$

where σ is Poisson's ratio, E and G being the stretch and shear moduli.

Let a be the radius and l the length of the shaft; put

$$r = au, z = av, l = ak.$$

If the only external stresses are shears q_1 perpendicular to radii in the end sections, the boundary conditions are, if $f(u)$ is any function of u,

$$\begin{array}{lll} q_1 = Gaf(u) & \text{at} & v = 0, k \\ q_1 = 0 & & u = 0 \quad (A) \\ q_3 = 0 & & u = 1 \\ p_1 = 0 & \text{at} & u = 1 \\ p_3 = 0 & & v = 0, k \quad (B) \\ q_2 = 0 & & v = 0, k \\ q_2 = 0 & & u = 1 \end{array}$$

When q_1 and q_3 are expressed in terms of η, (2) can be solved for η and conditions (A) are sufficient for finding the constants of integration. But q_2 and the normal stresses are independent of η and therefore of the boundary stresses (A). It is easily verified that $q_2 = p_1 = p_2 = p_3 = 0$ satisfy (1), (3) and (B). Consequently the terminal couples arising from $q_1 = Gaf(u)$ produce internal shears q_1 and q_3 but no normal stresses. This is not true in general; it is necessary that the shaft be circular, the boundary conditions independent of θ, and the strains small. In the absence of one or more of these restrictions, (1)–(7) contain extra terms that make all the stresses interdependent.

Equations (2) and (7) with the substitutions

$$r = au, z = av, \eta = a^2w$$

give the dimension-free equation

$$\frac{\partial^2 w}{\partial u^2} + \frac{1}{u}\frac{\partial w}{\partial u} - \frac{w}{u^2} + \frac{\partial^2 w}{\partial v^2} = 0. \tag{8}$$

The result of separating the variables by means of $w = UV$, where U is a function of u, and V is a function of v, is

$$\frac{1}{U}\left(\frac{d^2U}{du^2} + \frac{1}{u}\frac{dU}{du} - \frac{U}{u^2}\right) = -\frac{1}{V}\frac{d^2V}{dv^2} = -\lambda^2, \tag{9}$$

where λ is constant. Since the modulus λ occurs only in (4) there will be no danger of confusing it with the λ in (9).

For $\lambda = 0$ we get

$$\frac{d}{du}\left\{\frac{1}{u}\frac{d}{du}(Uu)\right\} = 0, \quad \frac{d^2V}{dv^2} = 0$$

whence

$$U = \alpha u + \beta u^{-1}, \quad V = c_1 v + c_2.$$

The constant β must be zero to prevent U from becoming infinite at $u = 0$; $c_2 = 0$ since there is to be no translation at $v = 0$. Hence

$$w = cuv, \text{ where } c = \alpha c_1, \tag{10}$$

is a partial solution of (8).

When λ is not zero (9) can be written

$$\frac{d^2U}{du^2} + \frac{1}{u}\frac{dU}{du} + \left(\lambda^2 - \frac{1}{u^2}\right)U = 0 \tag{11}$$

$$\frac{d^2V}{dv^2} = \lambda^2 V, \tag{12}$$

the solutions of which are

$$U = \alpha_1 J_1(\lambda u) + \beta_1 K_1(\lambda u) \tag{13}$$

$$V = \alpha \sinh \lambda v + \beta \cosh \lambda v \tag{14}$$

where J_1 and K_1 are Bessel functions of the first and second kinds. Since $K_1(\lambda u)$ becomes infinite at $u = 0$, the constant β must be zero to keep U finite. Hence the complete solution has the form

$$w = cuv + \sum_{n=1}^{\infty} A_n J_1(\lambda_n u)(\sinh \lambda_n v + B_n \cosh \lambda_n v). \tag{15}$$

The quantities c, λ_n, A_n and B_n remain to be found.

From the condition $q_3 = 0$ at $u = 1$ we get from (7) and (15)

$$\sum_{n=1}^{\infty} \lambda_n A_n \left\{\frac{dJ_1(\lambda_n)}{d\lambda_n} - \frac{J_1(\lambda_n)}{\lambda_n}\right\}(\sinh \lambda_n v + B_n \cosh \lambda_n v) = 0.$$

If this is to hold for all values of v

$$\lambda_n \frac{J_1(\lambda_n)}{d\lambda_n} = J_1(\lambda_n)$$

which can be transformed into

$$J_2(\lambda_n) = 0.$$

Hence the coefficients λ_n are the roots of $J_2(\lambda_n) = 0$; the root $\lambda_0 = 0$ is to be excluded as it is accounted for in the first term of (15).

From $q_1 = Gaf(u)$ at $v = 0$, and (7) and (15),

$$f(u) = cu + \sum_{n=1}^{\infty} \lambda_n A_n J_1(\lambda_n u). \tag{16}$$

Now if $f(u)$ is developable in the form

$$f(u) = B_0(u) + \sum_{n=1}^{\infty} b_n J_1(\lambda_n u) \tag{17}$$

it can be shown (Watson, *Theory of Bessel Functions*, §18·3) that

$$B_0(u) = 4u \int_0^1 t^2 f(t)dt \tag{18}$$

and

$$b_n = \frac{2\lambda_n^2 \int_0^1 tf(t)J_1(\lambda_n t)dt}{(\lambda_n^2 - 1)J_1^2(\lambda_n) + \lambda_n^2 \frac{dJ_1(\lambda_n)}{d\lambda_n}} \tag{19}$$

where (Watson, §3·2), since $J_2(\lambda_n) = 0$,

$$\frac{dJ_1(\lambda_n)}{d\lambda_n} = \frac{1}{2} J_0(\lambda_n).$$

The identity of (16) and (17) requires

$$cu = B_0(u),\ b_n = \lambda_n A_n. \tag{20}$$

From $q_1 = Gaf(u)$ at $v = k$, and (15)

$$f(u) = cu + \sum_{n=1}^{\infty} \lambda_n A_n J_1(\lambda_n u)(\cosh \lambda_n k + B_n \sinh \lambda_n k).$$

For this to be identical with (17) we find

$$B_n = \frac{1 - \cosh \lambda_n k}{\sinh \lambda_n k}. \tag{21}$$

If $f(u) = hu$, h being a known constant, the numerator in (19) becomes zero since $J_2(\lambda_n) = 0$; (20) gives $c = h$, and (15) reduces to $w = huv$ or $\eta = hrz$.

But η, the circumferential displacement at radius r, is given by $\eta = r\tau$ where τ = angle of twist; hence $\tau = hz$. This is Saint-Venant's result for a circular shaft and shows therefore that in this case (namely, $f(u) = hu$) the terminal couples produce no disturbance. The engineers' solution starts with the assumption that initially straight radii in any shaft section are undistorted by stress and that the stress varies directly as its radial distance. The assumption is strictly true when $f(u) = hu$; that is, $\tau = hz$ is independent of r, and all points on a given radius get the same angular displacement.

The linear relation $f(u) = hu$ does not obtain when the end of a shaft is held by keys or setscrews. Let us therefore investigate $f(u) = hu^3$, which is decidedly non-linear and has the advantage of making the numerator in (19) integrable in closed form. Calculations in this case show that for $k =$

2—a shaft of one diameter long—the angle of twist at a given radius plotted against $v = z/a$ is practically a straight line from $v = 0.5$ to $v = 1.5$. That is, the influence of the terminal moments extends only about half a radius from the ends. The total angle of twist comes out to be about 5% more than the value predicted by the simple formula. For a very long shaft the effect of the non-linear distribution of the stresses at the ends disappears at a distance of about one radius. Note that $B_n = -1$ for large k; then for large v, (15) reduces to its first term.

This discussion has been limited to terminal loads having circular symmetry about the shaft axis z. It has shown that a non-linear symmetrical stress distribution at the ends does not change the linear relation *derived* by Saint-Venant and *assumed* by engineers. It is reasonable to infer that any type of distribution, symmetrical or not, will also be without appreciable influence.

TRANSPLANTATION IN DROSOPHILA[1]

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The application of the method of transplantation, which has been so fruitful in the study of many biological problems, to genetically well known material, has advantages which are at once apparent to those who have thought about questions concerning the relation of genes to the developmental process. In this communication we wish to present, in a preliminary way, certain results obtained by the use of a technic which we have developed specifically for the transplantation of organs of *Drosophila*.

In short, this technic consists of the injection of the desired organ into the body cavity of the host by means of a specially designed glass micropipette. This pipette is used in the pipette-holder and with the syringe and metal capillary tube of the Chambers' micro-injection apparatus. The operation is carried out free-hand under a dissecting microscope. In practice, two persons coöperate using two dissecting microscopes, one inclined, arranged so that their fields coincide.

By means of this technic we have successfully transplanted larval ovaries as well as imaginal discs of eyes, antennae, wings and legs. In all cases we have used as hosts, flies in the larval stages. In particular cases, organs taken from pupae have been implanted in larvae. The transplantations can be made rapidly and a sufficient number of individuals

having the implanted organ reach maturity to make the technic efficient for experimental work.

Following Sturtevant's work[2] on the differentiation of vermilion eye color in elimination mosaics of *D. simulans* in which he showed that the gonads play a part, we were led to try transplants of larval ovaries in *D. melanogaster*. It was found, on dissection, that such implanted ovaries grow and differentiate normally. In preliminary breeding tests it was found that functional connections of the implanted ovary with a host oviduct may be established. A series of transplantations of ovaries were then made to determine (1) the frequency with which implanted ovaries differentiate and (2) the frequency with which functional connections are established. The results with regard to the frequency of differentiation, as determined by dissection of adults, are as follows:

NUMBER OF OVARIES IMPLANTED	SEX OF THE HOST	NUMBER OF OVARIES OBSERVED	NUMBER OF INDIVIDUALS
1	Female	1	2
		2	22
		3	57
	Male	0	2
		1	16
2	Female	3	2
		4	4
	Male	1	1
		2	2
3	Female	5	2

Considering only those cases in which one ovary was implanted, it is seen that of 99 adults, 73 had the implanted organ developed and differentiated.

In order to determine the frequency with which functional connections of the implanted ovary were established, ovaries of one genetic constitution were implanted in hosts which differed by at least one gene (homozygous in both cases). By mating to males recessive for this gene, the progeny coming from the implanted ovary were easily distinguished from those coming from a host ovary. Of females with one supernumerary ovary, progeny were obtained from 44. In 20 of these cases part of the progeny came from eggs originating in the implanted ovary. This shows clearly that an implanted ovary can compete with the normal ovaries in becoming attached to the genital ducts. On dissection it was found that two ovaries were attached and the third unattached to the oviducts. Hence it is clear that in the 20 cases in which fertile eggs were obtained from the implant, a normal ovary had been replaced. If the attachment of two of the three ovaries were a simple matter of chance, the implanted ovary would be expected to become attached in 67 per cent of the cases. It is not surprising that this frequency is not realized considering the several factors

which might put the implanted ovary at a disadvantage. In addition to the 20 cases of functional attachment mentioned above, there were 3 cases of functional attachment in which dissection showed only two ovaries. In one of these the dissection was somewhat questionable. In the other two we must assume that a normal ovary was completely replaced by the implanted one, probably because of injury to a normal ovary during the operation.

We have made a small experiment to see if *D. simulans* ovaries are able to differentiate and become attached when implanted in *D. melanogaster* hosts. Four melanogaster females in which simulans ovaries had been implanted were tested by mating to melanogaster males (genetic markers used as above) and all showed simulans-melanogaster hybrids among their progeny.

The possible uses to which this method of transplanting ovaries can be put are obviously several, among them (1) the production of interspecific hybrids where psychological or mechanical difficulties in mating prevent making them in the ordinary way, (2) the study of cases of intra- and interspecific sterility (the work of Bytinski-Salz[3] on Celerio species is a good example of this), (3) the more precise definition of the characteristics of cases of maternal inheritance and (4) the study of the part played by gonads in various physiological processes such as, for example, in the case referred to above, the differentiation of eye colors (Caspari's work[4] on transplantation of testicles in Ephestia provides an example of this).

Our most extensive series of studies of transplants have been concerned with eye color differentiation. We have investigated in a preliminary way most of the eye color mutants known in *D. melanogaster*. We have also shown that successful eye transplants can be made between species as different as *D. melanogaster* and *D. funebris*. In this report we can give only examples to illustrate the kind of studies that are being made.

Most of the eye transplants made have been between larvae at a stage of development shortly before pupation. Optic imaginal discs, with or without the antennal discs with which they are attached at this stage, are implanted into the body cavities of host larvae. In this position they grow and differentiate normally except that at maturity the implanted eyes are inverted, i.e., the curvature is reversed, the facets being on the inside, the pigment cells on the convex surface. (Failure of evagination of implanted imaginal discs has been observed in the case of legs and is probably characteristic of implanted wings as well.) That pigment differentiation, which is what we have been most interested in, is normal, is shown by many "control" transplants, i.e., eye discs of a given constitution implanted in hosts of the same constitution.

Our experiments on eye transplants have confirmed the conclusions drawn from studies of spontaneous mosaics[5] in that most eye transplants

are auto-differentiating with respect to pigmentation, i.e., they show the pigmentation characteristic of development in their normal tissue environment. Sturtevant has shown that vermilion eye color is, under certain conditions, not autonomous in its development in mosaics. In transplants it is likewise not autonomous; a vermilion (*v*) eye implanted in a wild type host develops the pigmentation characteristic of wild type.[6] By means of transplantation we have been able to study many combinations not easily obtained in natural mosaics and in this way have found that cinnabar (*cn*), an eye color phenotypically similar to vermilion, is not autonomous in its pigment differentiation. Two other eye color mutants, scarlet (*st*) and cardinal (*cd*) likewise phenotypically similar to vermilion, are, however, completely autonomous in their pigment development in all the combinations in which we have studied them. In the case of *v* or *cn* eyes implanted in a wild type host, there is obviously some kind of "host-implant influence" leading to a change in the reactions concerned in pigment formation. In a previous communication[7] we have considered the question of the relation of the *v* and *cn* host-implant influences. Briefly, we have shown that a *v* disc in a *cn* host gives a wild type eye, but that a *cn* disc in a *v* host gives a *cn* eye. From this it is clear that the host-implant influences in the two cases are not the same. But at the same time it is evident that we are not dealing with two independent influences, one controlled by the *v* gene, the other by the *cn* gene, for, if such were the case, the reciprocal transplants should give similar results, wild type in each case.

Given the behavior of *v* and *cn* implants in a wild type host, we have available a means of testing other mutants for the presence or absence of the two host-implant influences mentioned above. Thus by implanting a *v* eye disc in a host homozygous for another eye color mutant, we can determine whether or not this mutant is characterized by the presence or absence of the *v* host-implant influence. In a similar way, a test for the *cn* host-implant influence can be made. In this way we have been able to show that when a *v* eye is implanted in certain hosts (eye color mutants), *v* is autonomous in development. In all such cases studied so far, *v* and *cn* implants behave in the same way, for example, in a claret (*ca*) host, both *v* and *cn* implants are autonomous, in *st* or *cd* hosts they are both modified to wild type. This corroborates the conclusion drawn from reciprocal transplants between *v* and *cn* in indicating that the *v* and *cn* host-implant influences are genetically—and presumably chemically—closely related. At the same time these studies lead one to the conclusion, *a priori* probable, that the autonomous or non-autonomous nature of a given character in transplants is not necessarily an inherent property of the gene differentiating that character but may be determined by the genetic nature of the tissue environment. Other experiments show that

the genetic constitution of the implant itself as well as the relative stages of development of implant and host may also influence this property of autonomy or non-autonomy of the implant.

As a concluding example, we shall mention briefly, experiments made to establish the time in development at which vermilion pigmentation is irreversibly determined, i.e., after which a vermilion eye disc implanted in a wild type host will no longer give wild type pigmentation. By implanting eye discs from *v* pupae of successively older stages in wild type larvae almost ready to pupate (this procedure was used because of technical difficulties in implanting eye discs in pupae), it was shown that the characteristic host-implant influence operates, under these conditions, until very late stages, until about 48 hours after the formation of a puparium (at 25°C.). Shortly after this time, actual pigment can be seen in the eye.

Details of the above-mentioned experiments, as well as a description of the technic, will be published elsewhere.

[1] The work on which this report is based was done at the Institut de Biologie physico-chimique, Paris.

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[6] Ephrussi, Boris, and Beadle, G. W., *Comptes rendus acad. sci.*, **201**, 98 (1935).

[7] Beadle, G. W., and Ephrussi, Boris, *Ibid.*, **201**, 620 (1935).

THE TEMPERATURE-EFFECTIVE PERIOD OF THE SCUTE-1 PHENOTYPE

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Introduction.—The recently published papers of Child[1] have shown that for temperatures ranging from 14 to 31°C. there exists a temperature-effective period for the scute-1 phenotype which lies entirely within the latter half of the third larval instar period, when a mass of flies is concerned. For all bristles of the scute pattern, and at all temperatures tested, the effective period occupies the same relative position in the developmental period; and it is apparently restricted to the period between 89.3 and 96.8% of egg-larval development for any individual fly.

The study reported here is likewise on the temperature-effective period

of the scute-1 phenotype, and differs only in the technique used. The temperatures used in this experiment were 25° and 40° to 41°C. Developing larvae cannot usually live through more than five hours at 40° to 41°C.; but three-hour exposures are generally not excessively lethal.

The results of the study are interesting largely because they differ markedly from those of Child. A temperature-effective period appears which, for the ocellar bristles, extends through all the developmental periods tested, from the time of egg laying through the early pupal period. Other bristles considered also show effective periods which lie outside the time of Child's temperature-effective period.

Conditions and Methods.—Before being used in this study, the scute (*sc*) and vermilion carnation (*v car*) stocks were selected for "perfect" scute and normal bristle patterns, respectively, for six generations during the course of a preliminary experiment. They were continued in mass matings thereafter. Fifteen to twenty pair matings were used in the experiment to distribute as evenly as possible any modifying genes still present in the stocks. Examination of the control data reveals no significant variation in mean bristle numbers during the course of the experiment.

The food used was a standardized corn-meal-molasses-agar mixture, with two drops of a yeast solution added to each culture. It was contained in half-pint milk bottles, about an inch in a bottle.

The incubators were regulated to a constancy of at least 0.5°C. The control incubator was at 25° and the experimental incubator at 41° to 42°C. Experimental cultures were kept with the control cultures excepting during the exposure period. Temperatures of the culture during exposure were read from a thermometer inserted in the middle of the food, where the larvae tend to congregate. It took approximately an hour for the temperature to rise to 40°C. when the cultures were placed in front of the bulbs heating the ends of the incubator. The temperature was approximately a degree lower in the middle of the incubator, in spite of the air currents set up by the constantly running fan. It was possible therefore to shift the cultures around and keep them at approximately 40°C. Three cultures were exposed at a time, and the temperature was read from one of them in each case. Tests showed that there was frequently a variation of approximately ±0.5°C. in the three cultures, with extremes at approximately 39.5° and 41.5°C. in the total number of experimental cultures. A large majority of the cultures, however, had temperatures ranging from 40° to 41°C., exclusive of the first hour. The temperature within the exposed cultures remained from 0.5° to 1° lower than the incubator temperature during the course of the exposure period. The thermometers showed no such differences *inter se* when placed side by side in the incubator. No difference in culture-incubator temperatures was observed in the cultures at 25°C.

Egg-laying periods were from four to six hours long in the experimental series, and up to twelve hours, with a few of one and two days, in the control series. The general plan was to secure two or three sets of experimental cultures during the day, and a set of control cultures during the night. Each set of parents was used for six to eight days, after having aged two days. Each experimental series contained cultures of from three to nine sets of parent flies.

The cultures were exposed at different periods of development for different periods of time. The developmental period of a culture was computed

TABLE 1

NUMBER OF BRISTLES PRESENT PER 100 SCUTE ♂♂

PERIOD OF DEVELOPMENT	SERIES	HOURS OF EXPOSURE, 40° TO 41°C. "a"	1/2	1	1 1/2	2	2 1/2	3	TOTAL "a"−3
0 to 10 hours (egg)	Exper.	21		20		20		20	21.0
		±1		±2		±2		±15	±1.0
	Control	10.5		12.0		12.0		12.0	12.0
		±0.6		±0.7		±0.7		±0.7	±0.7
20 to 30 hours (late egg and early first instar)	Exper.	10.3	16.9	15.5	11.0	7.6	29.9	29.5	17.8
		±0.4	±0.7	±0.7	±0.6	±0.5	±0.9	±0.9	±0.2
	Control	9.2	10.2	10.8	10.6	9.7	11.2	10.6	10.2
		±0.4	±0.3	±0.3	±0.3	±0.3	±0.3	±0.3	±0.3
48 to 58 hours (first half of second instar)	Exper.	15.0	15.9	14.9	19.4	26.6	19.7	19.9	18.8
		±0.8	±0.7	±0.7	±0.8	±0.8	±0.8	±0.9	±0.3
	Control	9.9	9.2	9.4	9.6	9.2	9.4	9.6	9.4
		±0.4	±0.3	±0.3	±0.4	±0.3	±0.3	±0.4	±0.3
72 to 82 hours (first quarter of third instar)	Exper.	19.5	20.0	19.2	17.2	22.0	18.4	18.2	19.4
		±0.8	±1.0	±1.1	±1.0	±1.1	±1.0	±0.9	±0.4
	Control	9.2	9.7	10.2	9.8	9.8	9.7	9.5	9.7
		±0.3	±0.4	±0.4	±0.4	±0.4	±0.4	±0.4	±0.4
96 to 106 hours (third quarter of third instar)	Exper.	6.2	10.6	9.5	14.8	13.9	13.2	7.7	10.6
		±0.6	±0.9	±0.8	±1.0	±0.9	±0.9	±0.7	±0.3
	Control	9.2	9.2	9.2	9.6	9.4	9.3	10.6	9.3
		±0.3	±0.3	±0.3	±0.4	±0.3	±0.3	±0.4	±0.3
120 to 130 hours (early pupal)	Exper.	11.9		13.8		12.5		15.1	13.3
		±0.9		±0.9		±0.8		±1.0	±0.4
	Control	9.4		9.4		9.2		9.4	9.3
		±0.3		±0.3		±0.3		±0.3	±0.3

from the time the parents were placed in the culture until the time the culture was removed from the exposure to high temperature. It was so regulated that in each case a large majority of the larvae were within a ten-hour developmental period through the exposure period. The developmental periods used were: 0 to 10 hours, 20 to 30 hours, 48 to 58 hours, 72 to 82 hours, 96 to 106 hours and 120 to 130 hours of development. In terms of development (Dobzhansky and Duncan[2]), these periods represent approximately: the egg, the late egg and early first instar, the first half of the second instar, the first quarter of the third instar, the third

quarter of the third instar and the early pupal developmental periods, respectively. A majority of the larvae had pupated at 120 hours. Occasional ones had pupated at 96 hours, and probably were ones developing from eggs which had started development before they were laid. Most of the larvae pupated within a ten-hour period.

Cultures were exposed for $^1/_2$, 1, $1^1/_2$, 2, $2^1/_2$ and 3 hours beyond the first hour in which the temperature was rising to 40°C. For convenience, the first hour will be referred to as "*a*." In the case of the egg and pupal periods, exposures were limited to "*a*," 1-, 2- and 3-hour groups; but in the other four periods the half-hour exposure groups were also carried through.

The bristle counts were made on males only, the F_1 of *sc* females and *v car* males. Each male was classified on both sides for the presence of the following bristles: anterior scutellar (*as*), posterior scutellar (*ps*), anterior notopleural (*anp*), post-vertical (*pv*), ocellar (*oc*) and anterior plus middle orbital (*or*). These were classed singly and in all the combinations in which they appeared. As reported by Child, it was found impossible to distinguish the anterior and middle orbital bristles. These have been grouped together.

Presentation of the Data.—Control cultures were run throughout the course of the experiment. These numbered 297 cultures and 56,340 flies of which 27,669 were males. It was found that when the cultures were grouped according to the number of flies hatching per culture, the mean numbers of bristles present per 100 *sc* males varied significantly. In general, the mean numbers of the head bristles—*or*, *oc* and *pv*—decrease with an increasing mean number of flies hatching per culture, especially in the groups with the smaller numbers of flies hatching per culture; and the mean numbers of thoracic bristles—*anp*, *as* and *ps*—increase with an increasing mean number of flies hatching per culture, especially in the groups with the larger numbers of flies hatching per culture. Because of this effect of the number of larvae developing together in a culture upon the mean bristle number in *sc* males, the experimental data are compared with control data which represent mean bristle numbers that would have been expected had these experimental series been raised entirely at 25°C. and hatched the same mean numbers of flies per culture.

A total of 65,741 flies (32,060 males) were classified from the 600 cultures of the six experimental series. Only those data pertaining to the *oc* bristles are presented here. The other data will be presented in a later paper.

The accompanying table shows the number of *oc* bristles present per 100 *sc* males in the experimental series and in the corresponding controls. The data are given for each group in an experimental series, and for the totals of all the groups in each series. The "probable error" is included in each case.

In four of the six experimental series—the egg, the first half of the second instar, the first quarter of the third instar and the early pupal periods—the

mean numbers of *oc* bristles exceed the mean numbers in the control data in every group, and the differences between control and experimental series are clearly significant in the totals. The effect of the exposure was markedly lethal in the egg stage (the eggs did not hatch), so that the number of males observed was not so large as in the other series.

In the series exposed during the late egg and early first instar period the increase in mean *oc* bristle number is not uniform. The $^1/_2$- and 1-hour groups show an increase, and the $2^1/_2$- and 3-hour groups show a very great increase. The difference in all the groups totaled together is also large and statistically significant. In the series exposed during the third quarter of the third instar the increase seems to be limited to the $1^1/_2$-, 2- and $2^1/_2$-hour groups. The mean of these three groups together is significantly above the controls, and can be considered to establish a temperature effect even though the total for all the groups of this series is not certainly significantly above the controls.

The other bristles considered likewise showed non-uniform increases or decreases during these last two developmental periods, and reasonably uniform changes in the other four periods. This suggests a possible fundamental difference between the periods at the beginning and near the ending of the larval developmental period, and those periods entirely within the egg, larval or pupal developmental periods.

Discussion and Conclusions.—These data indicate clearly that the scute-1 phenotype has a temperature-effective period at 40° to 41°C. that extends at least from the time the egg is laid through the early pupal developmental period, a relatively long period, just as Child's data indicate clearly that for temperatures ranging from 14° to 31°C. the temperature-effective period is limited to a relatively short period late in the third larval instar. Other workers, using temperatures similar to those used by Child, have found more or less similarly limited temperature-effective periods for other phenotypic characters of Drosophila, including the mean number of bristles present in the Dichaete pattern, the size of vestigial wings and the mean number of facets in Bar eye and its alleles and heterozygotes. (See Child's paper for references.) In the light of this study it is legitimate to question the established limits of the temperature-effective periods for these other characters and the significance of any theories of development based upon limited temperature-effective periods. It becomes clearly evident that all the possibilities of experimental temperature technique must be tried before any character can be said definitely to have a temperature-effective period that is limited to one part of the developmental period.

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POLLEN-TUBE GROWTH OF TEN SPECIES OF DATURA IN INTERSPECIFIC POLLINATIONS

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We have in our cultures ten herbaceous species of Datura. Hybrids may be obtained from either of the two possible reciprocal pollinations between certain closely related species; they have been obtained from only one of the two possible reciprocal pollinations between others; and a large majority of the combinations have been unsuccessful. In many instances fruits will begin to set with abortive ovules or seeds that fail to germinate, so that we may presume that in such cases pollen-tube growth was successful. A great majority of the combinations, however, fail to set fruit. We have found it interesting to observe the behavior of the pollen tubes following cross-pollinations between the various species. The general experimental methods used in this investigation, as well as the methods used here in representing and evaluating our data, have been described elsewhere.[1]

The ten species offer material which is comparable in many important respects. Preliminary investigations begun more than twelve years ago had shown that under favorable conditions the pollen of any species studied will germinate on the stigmas of the remaining species, but there are differences in pollen-tube behavior.[2] All of these species have the same number of chromosomes ($n = 12$) and should polyploidy become involved, the difference due to chromosome number would be likely to show in the morphology and growth habits of the plants. Furthermore, we have investigated pollen-tube growth in polyploid plants of the species *D. stramonium*.[1,3]

In the size of the flowers and lengths of the pistils they show a wide range. Pistils of *D. leichhardtii* are about 30 mm. long, those of *D. quercifolia* about 40 mm. and those of *D. meteloides* about 190 mm. Their cultural requirements are sufficiently similar so that they may be grown together as greenhouse or out-of-door plants. We are, therefore, dealing with plants which represent ten species as they occur in nature. If we were to recognize some of the strains described as distinct species in the taxonomic literature there might be several more species to be considered. The forms *D. tatula* and *D. inermis*, for example, differ from *D. stramonium* only by single genes.

During the last five years we have made repeated efforts to obtain numerous tests of pollen-tube growth between all combinations among the

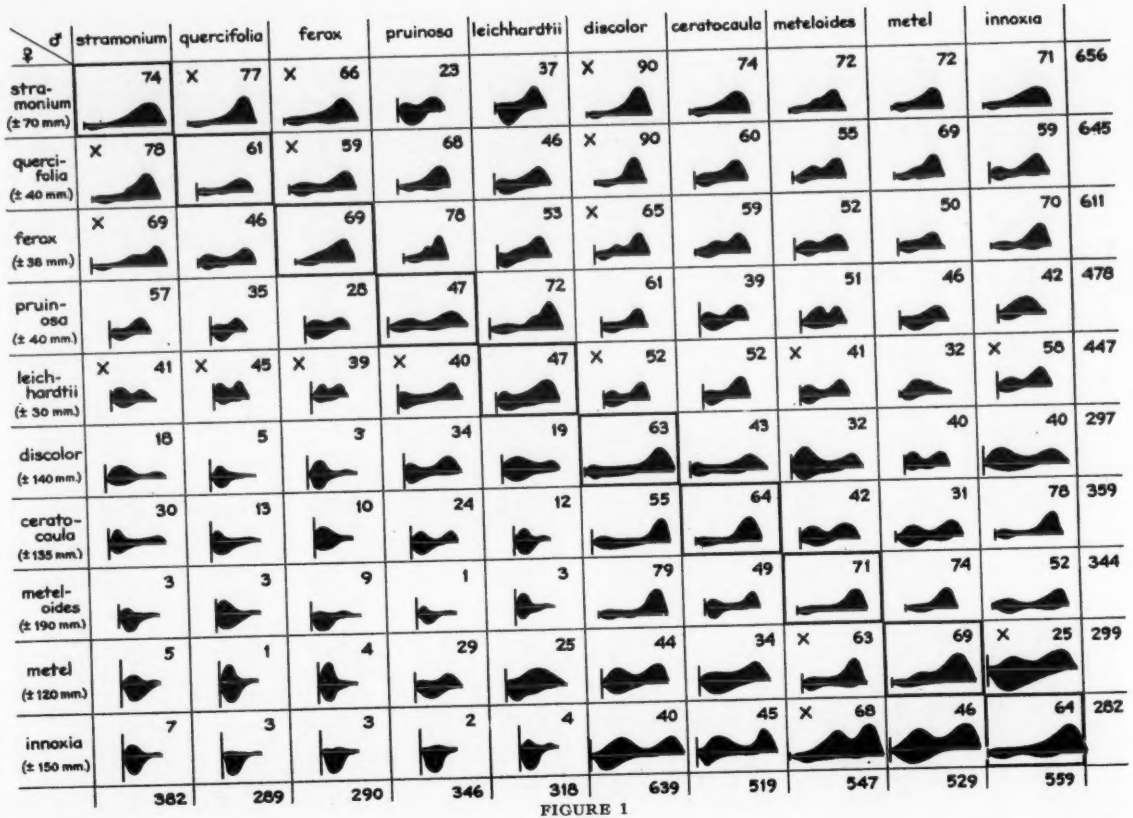

FIGURE 1

Diagrams of the pollen-tube distributions obtained in all possible crosses between the species of Datura. The lengths of pistils are shown under the names of species at the left.

ten species at times when the conditions of flowering and fruit setting appeared to be most favorable. Many of the experiments were repeated during the summer of 1935 also, with the assistance of Mr. H. C. Eyster. The results of these tests are summarized in the present paper.

The accompanying chart is a representation in miniature of the types of pollen-tube distribution diagrams obtained in each of the ten self- or sib-pollinations and in the ninety cross-pollinations which are possible between these ten species. The temperature during pollen-tube growth was 19°C. and the tests are based on an average of ten successful experiments for each kind of cross, not including our preliminary tests or any of the tests which were made at times when the plants were not in a suitable phase of their reproductive cycle. For a few of these tests we have obtained only four or more observations, for others we have included the average results observed in more than twenty tests, so that our chart represents the average results obtained in over a thousand individual experiments.

The histograms (Fig. 1) representing smoothed pollen-tube distribution curves were generalized as free-hand sketches, copied from the more favorable conditions which had been observed and were actually counted. This method of selecting the more favorable tests is justified, inasmuch as the fruits will set only from the pollinations made under the more favorable conditions, if they set at all, and it is well known that even in sib- and self-pollinations failures will be recorded.

The diagrams in our chart are represented with the length of the datum line proportional to rate of pollen-tube growth as observed at 19°C. In all of our studies of pollen-tube growth rate we have obtaineb a straight line relationship, except for the earliest period of pollen germination.[2] The species differ somewhat in the rate of their pollen-tube growth. All of the species will show a more or less progressive variation in rate for self- and sib-pollinations, depending upon the phase of their development in their reproductive cycle, so that since these rates are based on averages, some individual tests may at times go higher. Thus *Datura innoxia* with a rate of 3.3 mm. per hour at 19°C. when selfed (based on an average of 11 tests) has the highest pollen-tube growth rate, and *D. quercifolia* was observed to have the lowest selfed pollen-tube growth rate (1.9 mm. per hour at 19°C. based on an average of 15 tests). *D. ceratocaula* has a pollen-tube growth rate of only 2 mm. per hour (based on an average of 19 tests) and this value may be somewhat lower than its true rate since the rate observed is higher than this value when this pollen is used on the pistils of several other species—*D. innoxia, metel* and *discolor*. However, the values used here and for the base lines throughout the chart are the ones actually obtained from all of the tests which seemed to be satisfactory whether from self- or from cross-pollinations. The first flowers produced

Diagrams of the pollen-tube distributions obtained in all possible crosses between the species of Datura. The lengths of pistils are shown under the names of species at the left.

by a plant which is coming into the reproductive phase and some of the last ones after a plant becomes pot-bound or has experienced the effect of a lowered temperature or other adverse conditions, may be considerably below the values which were considered in our averages. Under such adverse conditions fruit will usually not set.

The figure placed in the upper right-hand side of each rectangle along with the diagram is the distribution index value. This is the percentage of the normal pollen tubes which had grown at least half the length of the longest pollen tubes. We consider this distribution value as a fairly useful index, one which gives us a measure of the probable success in pollen-tube growth and fertilization. We used this index in our analysis of balanced polyploids.[1] It represents a value which may be expected to be closely proportional to that part of the pollen-tube population which reaches the ovary and may participate in fertilization. The average distribution index value for the 18 combinations which have given hybrids is 59.

It may be noted that according to this analysis, some of the cross-pollinations are as good as or even better than the self-pollinations of either of the species represented in the cross. There is no question but that in these cases pollen-tube growth is highly satisfactory. However, if the reciprocal crosses of these with high index values are considered and compared, they are in most cases very unsatisfactory. Thus the cross *D. stramonium* or *D. quercifolia* pollinated with *D. meteloides*, *metel* or *innoxia* show very satisfactory conditions of pollen-tube growth, while the reciprocal pollinations belong to a class in which pollen-tube growth is so unsatisfactory that there could be no hope of obtaining any setting of fruits. Pollinations such as the last named have always failed in the field and on potted plants in the greenhouse. However, from some of these crosses with high distribution index values, fruits with abortive seeds may be obtained,[2] and furthermore it has been recently shown[4,5] that from one of these crosses zygotes begin to develop, but these zygotes do not exceed the eight-celled stage of the embryo.

Even though some of the crosses such as *D. stramonium* pollinated by *D. discolor* and by *D. innoxia* have high distribution index values as these are given here, the pollen-tube growth rate shown by the length of the distribution diagram is considerably below that of either of the species concerned when selfed. However, hybrids are obtained without difficulty from the cross *D. stramonium* × *D. discolor* where the two species have the rate when selfed of 2.9 mm. and 3.0 mm. per hour, respectively, and the rate in the cross-pollination is lowered to only 2.1 mm. per hour. When cross-pollinations with pollen-tube distributions of high index values similar to this one fail to give sets or viable seeds, the cause of this failure is not due to unsatisfactory conditions of pollen-tube growth.

However, in the reciprocal pollinations for which the distribution index is very low indeed, pollen-tube growth conditions are responsible for complete failure.

A cross (×) in a rectangle indicates that hybrids have been obtained. However, these do not include all possible hybrids. Some of these hybrids were obtained only after very many attempts, and the possibility of obtaining crosses between certain of the large-flowered species has not been thoroughly investigated. The short-styled species *leichhardtii* used as a female has yielded the largest number of hybrids. Usually the shorter style when employed as a female between two possible reciprocal pollinations is the more successful, both with respect to the hybrids which we have actually obtained and with respect to the pollen-tube growth conditions found in our tests. However, our tests show a total of 56 cross-pollinations having a pollen-tube growth index better than or as high as the lowest (*leichhardtii* × *ferox* 39%) from which hybrids have been obtained on the species with the shortest style. Furthermore, by the same standard, there appear to be only a few combinations among the 10 species in which fertilization may not be obtainable in one or the other of the two possible reciprocal pollinations.

The distribution index value placed in each rectangle may be used in ranking the styles of the ten species. By adding these values horizontally across the page we obtain totals that show us which styles may be considered the best general culture media for the pollens of all of the other species. The pistils of *D. stramonium*, *D. quercifolia* and *D. ferox* are the most favorable in this respect. The styles of the different species rank as follows: *stramonium*, *quercifolia*, *ferox*, *pruinosa*, *leichhardtii*, *ceratocaula*, *meteloides*, *metel*, *discolor*, *innoxia*.

If we add these index values in vertical columns we obtain a similar set of index totals for the comparison of the pollens of the different species. In this ranking, *D. discolor* stands highest. Its pollen is able to grow best on the pistils of the other nine species. The pollen ranks are: *D. discolor*, *innoxia*, *meteloides*, *metel*, *ceratocaula*, *stramonium*, *pruinosa*, *leichhardtii*, *ferox*, *quercifolia*.

It is a singular fact that the pistil rating of *D. discolor* is among the lowest though its pollen is the best when used on the other species, whereas *D. quercifolia*, which stands very near the top of the list for pistil quality, is at the bottom of the list with respect to pollen quality. *Innoxia*, which is at the bottom in pistil rating, stands second with respect to its pollen.

In summarizing the differences and resemblances of reciprocal crosses we are impressed by the necessity of taking into account the degree of morphological resemblance between the species. *Stramonium*, *quercifolia* and *ferox* constitute a sub-genus or a group within the genus; *pruinosa* and *leichhardtii* seem to constitute another similar group. The remaining

five may be more or less inter-related but they are probably not in a closely allied single group and may represent several sub-generic divisions.

With this grouping in mind we find that within the first of these groups, the *stramonium* group, there are no essential differences in the behavior of the pollen tubes in reciprocal crosses. The conditions are fairly good in either of a pair of reciprocal pollinations. If our investigation had been confined to the three species of this group the results would have shown essentially similar conditions of pollen-tube growth. The same is more or less true within the *pruinosa-leichhardtii* group except that there are distinct bimodal conditions in some of the combinations within this group as well as among the remaining species for which we can offer no explanation at present.

If, on the other hand, our investigation had happened to include only one species taken from the *stramonium* group, one from the *leichhardtii-pruinosa* division, and either *discolor* or *cerotacaula* plus one only of the remaining three, the conclusion might have been reached that reciprocal pollinations always show great differences in the behavior of the pollen tubes.

We believe that these findings, aside from being useful in attempts at hybridization, may be useful to the taxonomist, in supplementing the morphological characters which are used in arranging the probable phylogeny of the species within this genus. It is evident that the facts observed in interspecific pollinations have something to do with the natural relationships among these plants.

[1] Buchholz, J. T., and Blakeslee, A. F., *Genet.*, **14**, 538–568 (1929).
[2] Buchholz, J. T., and Blakeslee, A. F., *Mem. N. Y. Hort. Soc.*, **3**, 245–260 (1927).
[3] Buchholz, J. T., and Blakeslee, A. F., *Am. Jour. Bot.*, **19**, 604–626 (1932).
[4] Blakeslee, A. F., Murray, M. J., and Satina, S., *Records, Gen. Soc. Am.*, **3**, 33 (1934).
[5] Satina, S., and Blakeslee, A. F., *Bull. Torrey Bot. Club*, **62**, 301–312 (1935).

DENSITIES OF ROCKS CALCULATED FROM THEIR CHEMICAL ANALYSES

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Introduction.—Full understanding of magmatic differentiation, magmatic stoping, two-phase convection and isostasy depends upon knowledge of the densities of igneous rocks, both holocrystalline and molten. Some information has already been supplied by direct measurements as well as by experiments on thermal expansion, on change of volume with melting and on compressibility. However, vesicular material and partly glassy material rarely permit reliable determinations of specific gravity by the immersion method. Abundance of vesicles and glass is common in extrusive lavas, which, because of their chemical and mineralogical variety, include more than half of the igneous species named in petrography. Since many of these represent former liquid phases, the densities of the liquids constitute essential data for reasoning about the march of differentiation. In the year 1920 Iddings published a method of calculating from the chemical analysis of a rock the density of the same material if assumed to be completely crystallized, non-vesicular and at room temperature.[1] Allowing for thermal expansion and for increase of volume with change of state, the density at higher temperatures up to that where the rock is molten may be found. The values obtained by special experiments permit fairly accurate estimation of both of these modifying factors.

Although Iddings' method is in essence the only one available in many important problems, it has been little used by petrologists and geologists. In his paper on "Isostasy and Rock Density" Washington adopted the general principle, with a slightly different procedure in the calculation of density.[2]

Iddings tested his method by comparing the calculated and actually measured densities respectively found for individual specimens of holocrystalline rhyolite, porphyry, granite, gabbro, essexite, gabbro-diorite and hornblende picrite. In all but the last two cases the correspondence was close. However, the method seems important enough to warrant more elaborate testing. This has been done by the present writer, with results here set forth.

Method of Calculation.—The analyses used are those of nearly or quite fresh, holocrystalline, plutonic rocks. Most of the analytical statements were taken from Washington's Tables.[3] The choice was limited by the number of direct measurements of density recorded. The number is regrettably small, and it is to be hoped that in the future chemists will

regularly furnish specific gravities of their materials. Each of many analyses, including water, was reduced to a total of 100 per cent. The norm (the "standard" mineral composition) was then calculated. In other instances the norms stated in Washington's Tables were accepted and recomputed as if the corresponding analyses had been reduced to 100 per cent.

The percentage weight of each standard mineral was multiplied by the specific gravity of that mineral, as given by Iddings:

Quartz	= 2.65	Diopside	= 3.28	Magnetite	= 5.17
Orthoclase	= 2.54	Enstatite	= 3.18	Ilmenite	= 4.73
Albite	= 2.61	Hypersthene[a]	= 3.33	Hematite	= 5.22
Anorthite	= 2.77	Hypersthene[b]	= 3.53	Apatite	= 3.20
Leucite	= 2.48	Forsterite	= 3.21	Pyrite	= 5.03
Nephelite	= 2.62	Olivine	= 3.27–3.37	Fluorite	= 3.18
Corundum	= 4.00	Fayalite	= 4.00		

[a] FeO, 14; MgO, 28.
[b] FeO, 28; MgO, 14.

The sum of these products represents the normative density of the rock, supposed anhydrous. Correction for H_2O+ (arbitrarily assumed to enter as a free phase) gives a normative density almost exactly equaling the actual density.

Testing the Method.—Table 1 states the results, either for single specimens or for averages from calculations of a number of separate analyses. Obviously the larger this number is for each rock type the better the test. Column 4 of the table records the actually measured density, and column 5 the difference from the water-corrected normative density, in percentage of the measured density. It should be noted that throughout this paper the word "density" is, for convenience, used in the unconventional sense of the weight ratio between mineral or rock and water at room temperature. The reduction to the weight of equal volume of water at 4°C. would decrease each ratio by about 2 in the third decimal place.

Washington's suggestion that the mode of calculation should be changed in some details was based on good theoretical grounds, but actual trial shows the more arbitrary method here described to lead to a little better matching of normative density and measured density.

Each percentage difference in column 5 includes the algebraic sum of errors of measurement and computation, but is extremely close to that expected when allowance is made for the porosity of the rock specimen and for any real, though small, departure from perfect freshness. It thus appears that for each of the more abundant kinds of holocrystalline rocks (relatively poor in combined water and not containing excessive proportion of mica, hornblende, pyroxene or iron-rich olivine) the water-corrected

normative density may be regarded for practical purposes as the actual density.

Where the combined water is much in excess of one per cent by weight the calculated density is too small. Illustration is found in the remarkable "rhomb-porphyry" and associated "shackanite" at Rock Creek, British Columbia. The specific gravities of these fresh, non-vesicular rocks were determined by using whole hand specimens.[4] The normative densities

TABLE 1

CALCULATED AND MEASURED DENSITIES OF INDIVIDUAL SPECIMENS OF ROCK

ROCK TYPE	1 NUMBER OF SINGLE DETERMINATIONS	2 AVERAGE NORMATIVE DENSITY, ANHYDROUS	3 AVERAGE NORMATIVE DENSITY, CORRECTED FOR WATER	4 AVERAGE MEASURED DENSITY	5 EXCESS OF NORMATIVE DENSITY OVER MEASURED, IN PERCENTAGE OF THE LATTER
Granite	10	2.689	2.666	2.654	0.4
Quartz monzonite	6	2.770	2.743	2.733	0.4
Granodiorite	7	2.766	2.732	2.722	0.4
Quartz diorite	8	2.841	2.805	2.800	0.2
Diorite	7	2.865	2.829	2.809	0.6
Norite	10	3.005	2.984	2.980	0.1
Gabbro	10	3.050	3.000	2.985	0.5
Diabase	13	3.071	3.019	3.000	0.6
Dolerite	8	3.044	2.988	2.974	0.6
Essexite	6	2.964	2.915	2.881	1.2
Syenite	10	2.776	2.741	2.729	0.4
Monzonite	10	2.874	2.826	2.794	1.1
Nephelite syenite	8	2.697	2.653	2.640	0.5
Foyaite	3	2.700	2.666	2.640	0.6
Ijolite	5	3.022	2.950	2.991	−1.4
Anorthosite	6	2.810	2.793	2.769	0.9
Pyroxenite	5	3.278	3.244	3.264	−0.6
Websterite	2	3.261	3.190	3.302	−3.4
Wehrlite	1	3.181	3.172	3.370	−5.9
Lherzolite	1	3.234	3.219	3.330	−3.3
Mica peridotite	1	3.397	3.297	3.275	0.7
Fresh dunite	1	3.292	3.259	3.270	−0.3
Hortonolite dunite	1	3.616	3.580	3.752	−4.6
Hornblendite	1	3.366	3.234	3.288	−1.7
Titaniferous iron ore	1	4.355	4.263	4.138	3.0

of both the central phase (H_2O+ at 3.15 per cent) and contact phase (H_2O+ at 3.19 per cent) of the chonolithic porphyry are each about 2 per cent smaller than the respective measured densities of these rocks. For the shackanite (H_2O+ at 4.63 per cent) the difference is about 3 per cent in the same sense.

On the other hand, the method gives too low values for the densities of the peridotites in general, including the fresh South African hortonolite dunite, which is essentially composed of the densest known silicate.

A check on the method has been made by comparing normative and measured densities of plutonic species on the basis of world averages for the chemical analyses and world averages for measured densities. See table 2. The values in columns 5 and 6 of the table would be slightly different if more measured densities were published with analyses. The percentage differences of column 6 are naturally larger than those of column 5, table 1, because the world averages refer to a larger proportion of somewhat weathered specimens, with corresponding excess of water. The result of this second group of computations is to confirm the conclusion derived from the first group—that the method suggested by Iddings is sound when applied to the more abundant species of igneous rocks.

TABLE 2

WORLD AVERAGES OF NORMATIVE AND MEASURED DENSITIES

ROCK TYPE	1 NUMBER OF ANALYSES AVERAGED	2 NUMBER OF SINGLE DETERMINATIONS OF DENSITY	3 AVERAGE NORMATIVE DENSITY, ANHYDROUS	4 AVERAGE NORMATIVE DENSITY, CORRECTED FOR WATER	5 AVERAGE MEASURED DENSITY	6 EXCESS OF NORMATIVE DENSITY OVER MEASURED, IN PERCENTAGE OF THE LATTER
Granite	546	155	2.733	2.705	2.667	1.4
Quartz monzonite	20	10	2.743	2.714	2.698	0.6
Granodiorite	40	11	2.794	2.758	2.716	1.5
Quartz diorite	55	21	2.863	2.815	2.806	0.3
Diorite	70	13	2.932	2.877	2.839	1.3
Norite	24	11	3.057	3.021	2.984	1.2
Gabbro	50	27	3.053	3.010	2.976	1.1
Diabase	90	40	3.095	2.966	2.965	0.0
Dolerite	20	14	3.110	3.050	2.993	1.9
Essexite	20	11	3.035	2.976	2.937	1.3
Syenite	50	24	2.849	2.790	2.757	1.2
Monzonite	27	12	2.902	2.853	2.792	2.2
Ijolite	6	3	3.015	2.992	2.990	0.0
Anorthosite	12	12	2.778	2.742	2.734	0.3

Some Applications.—1. The method gives a somewhat useful control over estimate of the density of the most voluminous of all lavas, plateau basalt. According to its average chemical analysis as computed by the writer, the normative density of plateau basalt with one per cent of combined water is 3.071, which is thus a good value for the actual specific gravity of the same material when completely crystallized and non-vesicular. The value would be changed no more than a few units in the third place of decimals, if the holocrystalline rock be assumed to be 60 kilometers beneath the earth's surface; for the effect of compression on the volume of such material is almostly exactly annulled by that of thermal expansion. In the vitreous state the same material, at the depth of 60 kilometers, would have a density close to 2.795; at and near the earth's

surface, a density near 2.725—possibly as low as 2.700. Crystals of early-formed plagioclase would tend to rise in this liquid; crystals of pyroxene and olivine would tend to sink. A similar type of separating movement should have characterized the Non-porphyritic Central magma of the Island of Mull, but computation shows that bytownite crystals would have almost precisely the same density as the molten Porphyritic Central magma of the same island. Bowen has already discussed these relations in connection with the problem of magmatic differentiation.[5]

2. Since pore-free, holocrystalline phases of the Kilauean basalts of Hawaii are comparatively rare, it has seemed worth while to find the normative densities of some of these island lavas.

From Washington's chemical analyses[6] the following values were computed:

Average for 13 Kilauean basalts	3.131
Halemaumau lava dipped from crater in 1911	3.110
Pele's hair (glass)	3.054

The measured specific gravity of Pele's hair at room temperature is 2.850, a value which is 0.204, or 6.7 per cent smaller than that of the normative density. The percentage difference is intermediate between those experimentally determined for diabase by Day, Sosman and Hostetter and for dolerite by Douglas.[7]

3. On the average the chemical analysis of a plutonic type of rock is slightly more salic than that of the corresponding extrusive type. That the normative densities differ is illustrated by examples derived from world averages of chemical analyses (table 3).

TABLE 3

NORMATIVE DENSITIES OF PLUTONICS AND CORRESPONDING EXTRUSIVES

ROCK TYPE	NUMBER OF ANALYSES AVERAGED	NORMATIVE DENSITY, ANHYDROUS	NORMATIVE DENSITY, CORRECTED FOR WATER
Granite	546	2.733	2.705
Rhyolite	126	2.701	2.651
Granodiorite	40	2.794	2.758
Dacite	90	2.795	2.736
Diorite	70	2.932	2.877
Andesite	87	2.879	2.828
Syenite	50	2.849	2.790
Trachyte	48	2.776	2.734
Essexite	20	3.035	2.976
Trachydolerite	34	3.019	2.965

4. The method is capable of supplying a quantitative idea of the porosity of certain rocks. An example is found in the ideally fresh trachytes of Ascension Island.[8] Although these rocks lack bubble vesicles, they are charged with multitudes of submicroscopic, inter-feldspar pores. The specific gravities of two whole hand specimens were measured in the usual way, by immersion in water, and found to be 2.540 and 2.640. These values were manifestly too low, but slightly higher than would have resulted if the water had not penetrated the outer pores at all. Two trachytes without visible quartz gave respective normative densities of 2.683 and 2.718. The mean porosity is thus somewhat more than four per cent. Two other specimens containing micropoikilitic quartz gave norm densities of 2.678 and 2.723 and mean porosity again at about four per cent.

5. The specific gravity of bubble-free obsidian, forming a more salic phase of the Ascension trachyte, was determined directly at 2.415. The difference from the normative density, 2.624, is 7.6 per cent of the normative density, a reasonable value for the change from a holocrystalline condition to the vitreous, at room temperature.

METEORITE	NORMATIVE DENSITY	MEASURED DENSITY	PERCENTAGE DIFFERENCE
Mangwendi	3.482	3 517	−0.9
Fayette	3.466	3.51	−1.2
Nakhla	3.292	3.470	−5.1
Crumlin	3.700	3.553	4.1
Warbreccan	3.696	3.48	6.2
Strathmore	3.711	3.53	5.1
Soko-Banja	3.506	3.502	0.1
Witterkratz	3.688	3.49	5.7
Average for above	3.565	3.506	1.7

6. Another way in which the method has value appears when it is applied to the calculation of the mean density of the Pre-Cambrian complex of Finland at the surface, using Sederholm's weighted average analysis of these Sialic rocks.[9] The density comes out at 2.705, a value which may be used as fairly accurate for the material dominant near the surfaces of the continental masses.

7. The Iddings method has been applied to eight individual stony meteorites, with the locality names appearing in the following table of results.[10]

Considering the difficulty of perfect sampling of these stones both for chemical analysis and determination of average density, it is hardly surprising that the percentage differences shown in the last column are greater than the corresponding differences for the eruptive rocks.

The normative density computed from the average analysis of 20 achondritic meteorites[11] is 3.207. The actual mean density must be approximately the same. If so, it would closely match the mean density

of the Moon if our satellite were at atmospheric pressure throughout. This suggests a question for future investigation: is the Moon essentially composed of material identical with the average achondritic meteorite and crystalline nearly or quite to the center?

[1] J. P. Iddings, *Amer. Jour. Sci.*, **49**, 363 (1920).

[2] H. S. Washington, *Bull. Geol. Soc. America*, **33**, 375 (1922).

[3] H. S. Washington, Prof. Paper 99, *U. S. Geol. Survey* (1917).

[4] See R. A. Daly, Memoir 38, *Geol. Survey of Canada*, 401–415 (1912). It may be noted that the spelling of "shackanite" was derived from the name of a flag station of the Great Northern Railway, as drafted on the original manuscript map supplied to the writer for use in the field. The correct name for this station, obtained from the railway company after the writer's memoir was published, is "Syackan." However, it seems inexpedient to change the original name.

[5] N. L. Bowen, *The Evolution of the Igneous Rocks*, Princeton, 134 (1928).

[6] H. S. Washington, *Amer. Jour. Sci.*, **6**, 361 (1923).

[7] A. L. Day, R. B. Sosman and J. C. Hostetter, *Amer. Jour. Sci.*, **37**, 1 (1914); J. A. Douglas, *Quart. Jour. Geol. Soc. London*, **63**, 145 (1907).

[8] See R. A. Daly, *Proc. Amer. Acad. Arts Sci.*, **60**, 49 ff. (1925).

[9] J. J. Sederholm, *Bull.* **70**, *Comm. Géol. Finlande*, 1925; cf. R. A. Daly, *Igneous Rocks and the Depths of the Earth*, New York, 1933, p. 185, for the data.

[10] The statements of the analyses and measured densities appear in the papers here listed: G. P. Merrill, *Bull.* **94**, *U. S. Nat. Mus.*, **48**, 67 (1916); B. Lightfoot, A. M. Macgregor and E. Golding, *Miner. Mag.*, **24**, 1 (1935); L. Fletcher, *Ibid.*, **19**, 161 (1921); G. T. Prior, *Ibid.*, **18**, 8 (1916); W. F. P. McLintock and F. R. Ennos, *Ibid.*, **19**, 328 (1922); G. T. Prior, *Ibid.*, **18**, 23 (1916) and **17**, 31 (1913).

[11] H. S. Washington, *Amer. Jour. Sci.*, **9**, 362 (1925).

THE IDEAL-DECOMPOSITION OF RATIONAL PRIMES IN TERMS OF ABSOLUTE VALUES

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1. *Introduction.*—A classical problem of algebraic number theory is the following: Given a rational prime p and a field $R(\theta)$, where R is the field of rational numbers and the integer θ is a root of the irreducible equation $G(x) = 0$; to construct the prime ideal decomposition of p in $R(\theta)$.

A complete and natural solution of this problem can be given in terms of non-Archimedean absolute values. Briefly, each prime ideal factor of p corresponds to a value in the ring $R[x]$ of polynomials with rational coefficients, and these values can be found by a step-by-step process which gives all the usual properties of the corresponding prime ideals. This solution is constructive, in that the decomposition of a specified prime can be obtained

in a finite number of steps directly from the irreducible polynomial $G(x)$ defining the field. Furthermore, this solution can be obtained without using the unique decomposition theorem for ideals in the field. Finally, the form of this solution gives a simple interpretation for a number of known irreducibility theorems.

The advantage thus gained by using absolute values instead of prime ideals is another instance of the close relation between these two concepts. Other examples of this relation are Krull's[1] treatment of decomposition theorems and Deuring's arithmetic theory of Galois fields.[2]

2. *Prime Ideals as Values for Polynomial Rings.*—A non-Archimedean *value* V of a ring S is a function Va defined for $a \epsilon S$ such that

$$Va \text{ is a real number or } +\infty;$$

$$V(ab) = Va + Vb, \quad V(a+b) \geqq \text{Min}(Va, Vb). \tag{1}$$

If $Va = +\infty$ only for $a = 0$, we call V *finite*. If λ is any positive constant, then $Wa = \lambda . Va$ is another value, which is said to be *equivalent*[2] to V.

If P is a prime ideal in the field $R(\theta)$ given in §1, and if P^t is the highest power of P dividing a given integer α, then the function

$$W\alpha = t, \quad W(\alpha/\beta) = W\alpha - W\beta$$

is a value of $R(\theta)$. Every finite value is equivalent to one of the values obtained in this way.[3] The values of $R(\theta)$ can also be derived from the non-finite values of $R[x]$ by using the homomorphism of $R[x]$ to $R(\theta)$. Thus if V is a value of $R[x]$ for which $VG(x) = \infty$, then any two polynomials $f(x) \equiv g(x)$ modulo $G(x)$ will have the same value in V. This common value can be taken as the value of the corresponding number $f(\theta) = g(\theta)$. Hence

THEOREM 1. *Each prime ideal P of $R(\theta)$ corresponds to one and (except for equivalent values) only one finite value W of $R(\theta)$. Each value W corresponds in turn to one and only one of those values V of $R[x]$ for which $VG(x) = \infty$.*

3. *Inductive Values for Polynomials.*—However, each value V of the ring $R[x]$ can be built up in stages by the methods of an earlier note.[4] The value V_k on each stage is obtained from the V_{k-1} of the previous stage by assigning an augmented value μ_k to a suitable "key polynomial" $\phi = \phi_k(x)$. Since an arbitrary polynomial $f(x)$ can be expanded in powers of ϕ with coefficients $f_i = f_i(x)$ of degree less than ϕ, the value of $f(x)$ may be defined as

$$V_k[f_n\phi^n + \ldots + f_1\phi + f_0] = \text{Min}\,[V_{k-1}f_i + i\mu_k]. \tag{2}$$

In particular, the first stage V_1 is obtained in this fashion by using x for ϕ_1 and a value of R for V_0. Any V_k defined by (2) is called an inductive value.

Associated with any value V of a field there is a *value-group* Γ, consisting of all real numbers Va, and a *residue-class field*, which consists of the elements of non-negative value taken modulo the ideal of all elements of positive value. The residue-class field of an inductive value V_k has one of the forms[4] $F_k(y)$ or F_k, where F_k is an algebraic extension of the residue-class field F_0 for the value V_0. We call the degree $d(V_k)$ of F_k over F_0 the *inertial degree*[5] of V_k. The index $e(V_k)$ of the value group Γ_0 of V_0 in the value group Γ_k of V_k is the *branch-exponent* of V_k. These quantities can be calculated by the formulas

$$e(V_k) = \tau_k . e(V_{k-1}),\ d(V_k) = [d(V_{k-1}) \cdot \text{Degree } \phi_k]/[\tau_{k-1} \text{ Degree } \phi_{k-1}]$$

where τ_k is the smallest integer such that $\tau_k \mu_k \epsilon \Gamma_{k-1}$.

Each value V of $R[x]$ gives a sequence $V_0, V_1, V_2, \ldots$ of inductive values such that either $V = V_k$ for some k, or else Vf is Lim $V_k f$ for $k \longrightarrow \infty$. By requiring each V_k to be *homogeneous* in a certain sense, we can insure that each value V arises from only one such sequence. We wish to construct those values V which correspond to ideal factors P of the rational prime p. In the first place, this means that the sequence of inductive values must start with that value V_0 of R which corresponds to p (see Theorem 1). Secondly, $VG(x) = \infty$. This can be shown to require $G(x)$ to be "ambiguous" in each V_k—where a polynomial $f(x)$ is called ambiguous in V_k whenever the calculation of $V_k f(x)$ in (2) yields at least two distinct terms $f_i \phi^i$ and $f_j \phi^j$ with the same minimum value $V_k f$.

THEOREM 2. *Let the given prime p correspond to the value V_1 of R. Then the prime ideal factors of p can be formed if for every integer k we can construct for $R[x]$ all the homogeneous k-th stage inductive values V_k which start with V_0 and which leave $G(x)$ ambiguous.* A value V_k of the type here specified we call a *k-th approximant.*

4. *The Inductive Construction.*—Each approximant V_{k-1} gives rise to one or more k-th stage approximants. To find them we use the notion of equivalence ($\sim$):

$$f(x) \sim g(x) \text{ (in } V) \text{ if and only if } V[f(x) - g(x)] > Vf(x).$$

By a Euclidean algorithm it can be shown that $G(x)$ has a decomposition, unique up to equivalence, of the form

$$G(x) \sim u(x) \phi_{k-1}^{\beta} \psi_1(x)^{\beta_1} \cdots \psi_t(x)^{\beta_t} \text{ (in } V_{k-1}).$$

Here $u(x)$ is an "equivalence-unit," while the $\psi_i(x)$ are "equivalence-irreducible." This decomposition can be found in a finite number of steps. Each factor $\psi_i(x)$ can serve as a key polynomial for a new approximant V_k, while the possible values μ_k to be assigned to ψ_i can be determined by the slopes of the Newton polygon of $G(x)$ with respect to $\psi_i(x)$. All approximants V_k are obtained in this way.

The number of approximant V_k's may thus increase with k. This cannot occur if k is sufficiently large, for an analysis of the arithmetic properties of the discriminant of $G(x)$ gives the following essential finiteness theorem:

THEOREM 3. *There is an integer k' such that for $k > k'$ each approximant $V_{k'}$ gives rise to one and only one approximant V_k. Furthermore*

$$\sum e(V_k).d(V_k) = \text{Degree } G(x), \tag{3}$$

the sum being taken over the exponents and degrees of all these V_k.

The proof starts by assigning a "multiplicity" to every approximant V_k, and then shows, in connection with (3), that these multiplicities eventually become unity. The argument indicates that we may take k' less than $n + \delta n \log n$, where δ is the power of p dividing the discriminant of $G(x)$. These results connect with the prime ideals as follows:

THEOREM 4. *The set of k'-stage approximant values $V_{k'}$ can be constructed in a finite number of steps. Each prime ideal factor P of p corresponds to one and only one $V_{k'}$. Furthermore, the degree of P is the inertial degree of the corresponding $V_{k'}$, while the branch exponent of $V_{k'}$ is precisely the power to which P divides p.*

The form of the decomposition of p is thus known, and the equation (3) contains a proof of the well-known expression for the degree of G. By choosing k sufficiently large and computing the value of each key polynomial $\phi_k(x)$ in every approximant value, it is also possible to constructively represent each ideal P as the greatest common divisor of principal ideals.

This method of decomposing p is closely related to a method due to Ore,[6] which, although formulated in ideal-theoretic language, is essentially equivalent to most of the first three stages of the above method. Because of this limitation of k, Ore's method is not applicable to all equations. A later method[7] of Ore's makes indirect use of certain higher stages to transform the defining equation.

5. *Irreducibility Criteria.*—Our construction method makes it possible to give a simple form to a number of arithmetic criteria for the irreducibility of a polynomial $H(x)$ with rational coefficients and non-vanishing discriminant. The general test is as follows: Replace $G(x)$ in the above investigation by $H(x)$ and calculate the k-th stage approximants for $H(x)$ and some p. If for some k there is but one approximant V_k, and if this V_k has the property

$$e(V_k).d(V_k) = \text{Degree } H(x),$$

then $H(x)$ is irreducible. In other words, treat $H(x)$ as if it were the defining equation of a field. If in this hypothetical field p appears to be a power of a single prime ideal, then $H(x)$ is actually irreducible! This short statement, which can be readily proved, includes the criteria of Schöne-

mann, Eisenstein, Dumas,[8] Ore,[9] Mac Lane[4] and others. It can be generalized to give theorems about the possible degrees of the factors of a reducible $H(x)$.

The details and proofs for this method of prime ideal decomposition will be published in a mathematical journal.

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[3] E. Artin, "Ueber die Bewertungen algebraischer Zahlkörper," *Journal f. d. Math.*, **167**, 157–159 (1932).

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[5] A. Ostrowski, "Untersuchungen zur arithmetischen Theorie der Körper," *Math. Zeit.*, **39**, 269–404 (1934).

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ON PSEUDO-LINEAR TRANSFORMATIONS

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The present note contains the principal results of a paper which I hope to publish in detail at some later date.

1. Let $\mathfrak{R}$ be a vector space of finite dimension n over an arbitrary field $\mathfrak{F}$ (not necessarily commutative). A *pseudo-linear transformation* (p. l. t.) $\mathbf{T}$ of $\mathfrak{R}$ is defined by the conditions

$$(x + y)\mathbf{T} = x\mathbf{T} + y\mathbf{T} \qquad x, y \in \mathfrak{R} \qquad (1)$$

$$(x\alpha)\mathbf{T} = (x\mathbf{T})\bar{\alpha} + x\alpha' \qquad \alpha, \bar{\alpha}, \alpha' \in \mathfrak{F} \qquad (2)$$

where the correspondence $\alpha \longrightarrow \bar{\alpha}$ is a 1-*automorphism* and $\alpha \longrightarrow \alpha'$ a *differentiation* in $\mathfrak{F}$, i.e.,

$$\overline{(\alpha + \beta)} = \bar{\alpha} + \bar{\beta} \qquad \overline{(\alpha\beta)} = \bar{\alpha}\,\bar{\beta} \qquad (3)$$

$$(\alpha + \beta)' = \alpha' + \beta' \qquad (\alpha\beta)' = \alpha'\bar{\beta} + \alpha\beta' \qquad (4)$$

and $\alpha \longrightarrow \bar{\alpha}$ is $(1 - 1)$.

The importance of these transformations is indicated by the following examples:

(*a*) $\mathfrak{F}$, any field; $\bar{\alpha} \equiv \alpha$; $\alpha' \equiv 0$. In this case $\mathbf{T}$ is an ordinary linear transformation of a vector space over a non-commutative field.

(*b*) $\mathfrak{F}$, the complex field; $\bar{\alpha}$, the conjugate complex of α; $\alpha' \equiv 0$. $\mathbf{T}$ is then an *anti-linear transformation* as defined by v. d. Waerden.[1] Such transformations are on almost an equal footing with linear transformations in complex projective geometry. We may generalize this type and obtain *semi-linear transformations*, i.e., p. l. t. for which $\mathfrak{F}$ is any field, $\alpha \longrightarrow \bar{\alpha}$ any 1-automorphism, $\alpha' \equiv 0$. Another instance of these can be found in linear difference equations (cf. (*c*)).

(*c*) The differential equations

$$u_i' \equiv \frac{du_i}{d\tau} = \sum_{j=1}^{n} u_j \alpha_{ji}(\tau) \quad (i = 1, \ldots n) \tag{5}$$

where the α's are analytic in τ, define a p. l. t. in the following way: Let $\mathfrak{R}$ be the n-space of forms $u = \sum u_i \alpha_i(\tau)$ in the independent vectors u_i with coefficients $\alpha_i(\tau)$ in the field of analytic functions of τ. We may associate with (5) the transformation $\mathbf{T}$ of $\mathfrak{R}$ defined by

$$u \longrightarrow u\mathbf{T} = \sum u_i \alpha_i'(\tau) + \sum u_i' \alpha_i(\tau)$$

where u_i' is given by (5). $\mathbf{T}$ is p. l.

2. If $(e_1, \ldots, e_n)$ is a basis for $\mathfrak{R}$, $\mathbf{T}$ is determined by its effect on the e_i. Suppose $e_i\mathbf{T} = \sum e_j \tau_{ji}$ or

$$(e_1, \ldots, e_n)\mathbf{T} = (e_1, \ldots, e_n)T \qquad T = (\tau_{ij}).$$

T is called *the matrix of* $\mathbf{T}$ in $(e_1, \ldots, e_n)$.

Let $\mathfrak{F}[\mathbf{T}]$ denote the ring of transformations in $\mathfrak{R}$ generated by $\mathbf{T}$ and the elements of $\mathfrak{F}$. The p. l. t. $\mathbf{T}_1$ and $\mathbf{T}_2$ with the same automorphism and differentiation are *similar* if the abelian group $\mathfrak{R}$ with the operators $\mathfrak{F}[\mathbf{T}_1]$, $(\mathfrak{R}, \mathfrak{F}[\mathbf{T}_1])$, is operator isomorphic with $(\mathfrak{R}, \mathfrak{F}[\mathbf{T}_2])$. This means that the matrices T_i of $\mathbf{T}_i$ are related thus:

$$T_2 = A^{-1}T_1\bar{A} + A^{-1}A'$$

where $A = (\alpha_{ij})$, $\bar{A} = (\bar{\alpha}_{ij})$, $A' = (\alpha'_{ij})$. Similarly we may employ the concept of group with operators to define *reducibility*, *decomposability*, *complete reducibility* of p. l. t.

It is well known that if $\mathfrak{F}$ is commutative and $\mathbf{T}_i$ $(i = 1, 2)$ are linear, then $\mathbf{T}_1$ and $\mathbf{T}_2$ are similar if and only if certain polynomials—the invariant factors or the elementary divisors—determined by them are the same. We prove an analogous result here. The method is briefly this: We introduce the ring $\mathfrak{F}[t]$ of *non-commutative polynomials*[2] in the indeterminate t where

$$\alpha t = t\bar{\alpha} + \alpha' \tag{6}$$

and the space $\mathfrak{R}[t]$ of all forms $\sum e_i\alpha_i(t)$, $\alpha_i(t)\epsilon\mathfrak{F}[t]$. $\mathfrak{R}[t]$ is a group $(\mathfrak{R}[t], \mathfrak{F}[t])$ with operators $\mathfrak{F}[t]$. The correspondences $\sum e_i\alpha_i(t) \longrightarrow \sum e_i\alpha_i(\mathbf{T})$, $\beta(t) \longrightarrow \beta(\mathbf{T})$ define an operator homomorphism of $(\mathfrak{R}[t], \mathfrak{F}[t])$ and $(\mathfrak{R}, \mathfrak{F}[\mathbf{T}])$ By the usual method[3] of reducing a matrix with elements in $\mathfrak{F}[t]$ to diagonal form by elementary transformations we may obtain a basis for $\mathfrak{R}$ such that the matrix of $\mathbf{T}$ has the form

$$\begin{pmatrix} \boldsymbol{T}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{T}_2 & & \vdots \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & \boldsymbol{T}_r \end{pmatrix} \tag{7}$$

where

$$\boldsymbol{T}_i = \begin{pmatrix} 0 & 0 & \cdots & \tau_{ni}^{(i)} \\ 1 & 0 & & \\ \vdots & \ddots & \ddots & \\ & & 0 & \\ 0 & \cdots & 1 & \tau_1^{(i)} \end{pmatrix} \tag{8}$$

The invariant subspace $(\mathfrak{R}_i, \mathfrak{F}[\mathbf{T}])$ corresponding to $\mathbf{T}_i$ is said to be *cyclic* and $\mathbf{T}$ acting in it, *cyclic* also. $(\mathfrak{R}_i, \mathfrak{F}[\mathbf{T}])$ is generated by a single vector g_i and its transforms $g_i\alpha_i(\mathbf{T})$ by the elements of $\mathfrak{F}[\mathbf{T}]$. The polynomial $\mu_i(t) = t^{n_i} - t^{n_i-1}\tau_1^{(i)} - \cdots - \tau_{n_i}^{(i)}$ is called the *order* of g_i. We have

THEOREM 1. *$(\mathfrak{R}, \mathfrak{F}[\mathbf{T}])$ is a direct sum of the cyclic subspaces $(\mathfrak{R}_i, \mathfrak{F}[\mathbf{T}]) \equiv (g_i)$ where the order $\mu_i(t)$ of g_i is both a right factor and a left factor of $\mu_j(t)$ for $j > i$.*

THEOREM 2. *Two cyclic subspaces are operator isomorphic if and only if their generators are similar in the sense of Ore.*[4]

The undecomposable parts of $(\mathfrak{R}, \mathfrak{F}[\mathbf{T}])$ are cyclic. The orders of the generators of these parts will be called the *elementary divisors* of $\mathbf{T}$. With the aid of a theorem of Krull[5] we may prove

THEOREM 3. *The p. l. t. $\mathbf{T}_1$ and $\mathbf{T}_2$ are similar if and only if their elementary divisors may be paired off into similar pairs.*

3. The orders $\mu_i(t)$ in Theorem 1 are not unique in general (even in the sense of similarity). Nevertheless they give criteria for the reducibility, decomposability and full reducibility of $\mathbf{T}$.

An extension of Theorem 1 may be given. To describe this and for other purposes we require the following definitions: A polynomial $\mu^*(t)$ is *finite* if the left ideal generated by it is two sided. It follows for our particular ring that the right ideal generated by $\mu^*(t)$ is also two sided. $\mu(t)$ is *bounded* if it is a left factor of a finite polynomial $\mu^*(t)$. It follows that $\mu(t)$ is also a right factor of $\mu^*(t)$. The finite $\mu^*(t)$ of minimum degree and leading coefficient 1 of which the bounded $\mu(t)$ is a factor is called *the bound* of $\mu(t)$. It is unique. A p. l. t. $\mathbf{T}$ is *finite* if $\mathfrak{F}[\mathbf{T}]$ has a finite basis over $\mathfrak{F}$. We prove

THEOREM 4. *The finite polynomials with leading coefficients one constitute a commutative system* $\mathfrak{B}^*$ *closed under multiplication and in which unique factorization holds.*

THEOREM 5. $\mathbf{T}$ *is finite if and only if the* $\mu_i(t)$ *are bounded.*

If $\mathbf{T}$ is finite, the bound $\mu_r^*(t)$ of the last $\mu_i(t)$ is called the *minimum function* of $\mathbf{T}$. It is analogous to the minimum function of an ordinary linear transformation. We have for instance

THEOREM 6. *A finite p. l. t. is completely reducible if and only if its minimum function has no multiple factors in* $\mathfrak{B}^*$.

The following extension of Theorem 1 may be proved.

THEOREM 7. *The invariant subspaces in Theorem* 1 *may be chosen so that each* $\mu_i(t)$, $i < r$, *is bounded and* $\mu_i^*(t)$ *is a factor of* $\mu_j(t)$ *for* $j > i$.

In most cases the $\mu_i(t)$ thus normalized are unique to within similarity. However, I have been unable to prove this as yet for the general case.

4. We consider next the automorphisms of $(\mathfrak{R}, \mathfrak{F}[\mathbf{T}])$. These give rise to matrices A such that $A\mathbf{T} = \mathbf{T}\bar{A} + A'$ and conversely. For cyclic transformations the ring of automorphisms $\mathfrak{A}$ is isomorphic to the invariant ring[6] of its order. The general theorems of v. d. Waerden,[7] Fitting[8] and others are applicable here and give the structure of certain hypercomplex systems. My former results[9] on cyclic algebras are obtainable in this way.

5. Non-trivial information about $\mathbf{T}$ can be obtained just from the nature of $\mathfrak{F}[t]$.

If by a suitable choice of t we may obtain $\alpha' \equiv 0$, $\mathfrak{F}[t]$ is called *semi-linear*, and if by a suitable t we obtain $\bar{\alpha} \equiv \alpha$, $\mathfrak{F}[t]$ is called *differential.*

THEOREM 8. *If* $\mathfrak{F}$ *is commutative,* $\mathfrak{F}[t]$ *is either semi-linear or differential.*

It is not difficult to determine the finite elements of all differential and semi-linear rings. In most cases these consist of the elements of $\mathfrak{F}$ alone. It follows that *the corresponding p. l. t. are cyclic.* There are also interesting special rings $\mathfrak{F}[t]$ in which every element is bounded. *The corresponding p. l. t. are finite.*

There are numerous applications of these results to complex projective geometry, differential and difference equation and hypercomplex numbers. I shall give these in the subsequent fuller exposition of this subject.

[1] v. d. Waerden, "Gruppen von lineare Transformationen," p. 4.

[2] Ore, "Theory of Non-Commutative Polynomials," *Ann. Math.*, **34**, 480–508 (1933).

[3] v. d. Waerden, *Moderne Algebra II*, pp. 135–142.

[4] Ore, loc. cit. in footnote 2, p. 488.

[5] Krull, "Über verallgemeinerte Abelsche Gruppen," *Math. Zeits.*, **23**, 182–187 (1925).

[6] Ore, "Formale Theorie der linearen Differentialgleichungen II," *Crelle Jour.*, **168**, 241–243 (1932).

[7] v. d. Waerden, *Moderne Algebra II*, pp. 165–169.

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FORMULAS GIVING THE NUMBER OF THE GROUPS DETERMINED BY SQUARES

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It is sometimes convenient to use formulas expressing the total number of the groups of given fixed orders which satisfy certain given conditions. Unfortunately the list of such known formulas is still very small but it may be desirable to provide such a partial list, especially since it is not always directly evident that some of these formulas are involved in known developments. In the present article we shall confine our attention to the special cases in which these formulas relate to the subgroups generated by the squares of the operators of a given group, beginning with the case when the orders of the groups under consideration are of the form 2^m. When $m < 7$ these groups have been listed and hence we shall consider here only the cases when m can have an arbitrary value but when the given properties of the groups are sufficiently restrictive to determine general formulas for the number of the possible groups.

Let G represent a group of order 2^m and let H represent the subgroup of G generated by the squares of its operators. It is directly evident that H is composed of those operators of G which have the property that none of them appears in the possible sets of independent generators of G; that is, H is the φ-subgroup of G. It is known that this subgroup is the cross-cut of all the subgroups of index 2 contained in G. A necessary and sufficient condition that H is of index 2 under G is that G is cyclic and when H is cyclic and of index 4 under G it is known that G is one of five elementary groups whenever $m > 3$ since G then contains a cyclic subgroup of index 2. The quotient group G/H is abelian and of type $(1, 1, 1, \ldots)$, and when H is of order 2 G belongs to one of three elementary categories of groups. The total number of distinct groups in all of these categories is either

$$3(m - 1)/2 \text{ or } (3m - 4)/2$$

as m is odd or even, respectively. These groups are also characterized by the fact that each of them contains only operators of order 4 and 2 besides the identity and that all of its operators of order 4 have a common square.

The three given categories of groups may be distinguished by the relative number of operators of order 4 in the groups composing the various categories. In one of these this number is obviously 2^{m-1} since each of its groups contains an invariant operator of order 4. To determine this number for the other two categories it is desirable to restrict the attention to the

case when G is not a direct product of two groups such that one of them is the abelian group of type (1, 1, 1, ...). In the category in which the number of the operators of order 4 in each group is less than half its order this number is then $2^{m-1} - 2^{(m-1)/2}$ while in the remaining category it is $2^{m-1} + 2^{(m-1)/2}$. These formulas result from the fact that if we represent in the former case by 2^α the order of the abelian group at which we arrive by successively finding the subgroup composed of the operators which are commutative with a non-invariant operator of order 2 in G it results that $\alpha = (m+1)/2$, and hence $(2^m - 2^\alpha)/2$ is the former result. If 2^α represents the order of the subgroup generated by the operators of order 2 in the corresponding Hamiltonian group in the latter case then $\alpha = (m-1)/2$ and hence $(2^m - 2^{\alpha+2})/2 + 3 \cdot 2^\alpha$ is the latter result.

When G is the four group two cases present themselves. In one of these the commutator subgroup of G is either the identity or of order 2, while in the other this commutator subgroup is the four group. The latter case seems very much more difficult than the former and has not as yet given rise to results which can be represented by convenient general formulas. In the former case, the number of the possible groups is expressed by the formulas[2]

$$3(m-3)/2 + 2, \; m \text{ odd, and } 3(m-4)/2 + 3, \; m \text{ even}$$

To exhibit the usefulness of these formulas it should be noted that they suffice to establish the fact that there are exactly 14 groups of order 16 since a group of this order whose squares constitute the four group cannot have a commutator subgroup of order 4. On the contrary, a group of order 32 may have the four group both as the group of its squares and also as its commutator subgroup, and hence not all of these groups are covered by the given formulas.

When H is the cyclic group of order 2^α, $\alpha > 1$, there are again two cases to be considered. In one of these the operators of H are invariant under G while in the other these operators are also transformed into their inverses thereunder. The total number of these groups is expressed by the following formulas:[3]

$$7(m-\alpha-1)/2 + 1 \text{ and } 7(m-\alpha)/2 - 2$$

as $m - \alpha$ is odd or even, respectively. For instance, there are 5 and only 5 groups of order 16 which have the cyclic group of order 4 as the group of the squares of their operators. While there is obviously no upper limit to the number of the groups which have a given group as the group of the squares of their operators, the condition that the order of such a group is a fixed number imposes not only such an upper limit but enables us to give the exact number of possible groups as is evidenced by the formulas given above.

We proceed to consider some formulas for the number of the possible

groups of a given order when the order of G is not of the form 2^m. Suppose first that H is a cyclic group of odd order h and that h is divisible by exactly n distinct prime numbers, while the order of G is divisible by 2^m but by no higher power of 2. The order of G is therefore $h \cdot 2^m$. To determine the number of the groups of this order it is convenient first to consider those groups which have H as the group of the squares of their operators but are not the direct product of such a group and an abelian group of order 2^k and of type $(1, 1, 1, \ldots)$. The number of these groups of order $h \cdot 2^\alpha$ is represented by the formula

$$1 + \frac{(2^n - 1)(2^n - 2) \ldots (2^n - 2^{\alpha-1})}{(2^\alpha - 1)(2^\alpha - 2) \ldots (2^\alpha - 2^{\alpha-1})}$$

If we include direct products in which one of the factors is the abelian group of order 2^k and of type $(1, 1, 1, \ldots)$ the number of these groups is represented by the formula

$$1 + \sum_{\alpha=1}^{\alpha=\alpha} \frac{(2^n - 1)(2^n - 2) \ldots (2^n - 2^{\alpha-1})}{(2^\alpha - 1)(2^\alpha - 2) \ldots (2^\alpha - 2^{\alpha-1})}$$

provided $\alpha \leqq n$. Hence it results that when $m \geqq n$ the number of these groups of order $h \cdot 2^m$ is expressed by the formula

$$1 + \sum_{\alpha=1}^{\alpha=n} \frac{(2^n - 1)(2^n - 2) \ldots (2^n - 2^{\alpha-1})}{(2^\alpha - 1)(2^\alpha - 2) \ldots (2^\alpha - 2^{\alpha-1})}$$

while when $m < n$ the number of the groups G is given by the formula

$$1 + \sum_{\alpha=1}^{\alpha=m} \frac{(2^n - 1)(2^n - 2) \ldots (2^n - 2^{\alpha-1})}{(2^\alpha - 1)(2^\alpha - 2) \ldots (2^\alpha - 2^{\alpha-1})}$$

When H is a cyclic group of general order it is known that the holomorph of H is the direct product of the holomorphs of its cyclic subgroups, and hence G can be constructed by establishing an isomorphism between the groups considered above. When H is a non-abelian dihedral group its order is not divisible by 8 and each of the odd prime factors of h is congruent to unity modulo 4. Hence it results that there are groups which cannot be the subgroup composed of the squares of the operators of a group. It may be proved as follows that the quaternion group has also this property. If H would be the quaternion group G would involve an operator of order 8 whose square would be an operator of order 4 in H. The former operator would transform two of the subgroups of order 4 in H into each other since its square would transform their generators into their inverses. It therefore results that G would involve an operator which would transform the three subgroups of order 4 in H according to an operator whose order is divisible by 3 and hence the order of G would be divisible by 3. That is,

there is no group which has the quaternion group as the group generated by the squares of its operators.

No complete group can be the group generated by the squares of the operators of a given group since the operators of this group would have to transform the operators of this complete group according to outer isomorphisms which is impossible. In particular, no symmetric group is the group of the squares of a given group because the symmetric group of degree 6 is the only symmetric group which is not complete and this is obviously not the group generated by the squares of the operators of a given group. The alternating group of degree m is clearly the group generated by the squares of the symmetric group of this degree as well as the group generated by the squares of its own operators.

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[3] G. A. Miller, *Proc. Nat. Acad. Sci.*, **20**, 372–375 (1934).

ON THE ZEROS OF THE SUCCESSIVE DERIVATIVES OF INTEGRAL FUNCTIONS

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Let $\eta_0, \eta_1, \eta_2 \ldots$ be a sequence of complex numbers and $f(x)$ an analytic function. The expansion in series of polynomials

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(\eta_n)\, p_n(x), \tag{1}$$

where

$$p_n(x) = \int_{\eta_0}^{x} dx_1 \int_{\eta_1}^{x_1} dx_2 \ldots \int_{\eta_{n-1}}^{x_{n-1}} dx_n \qquad (n \geqq 1,\ p_0(x) = 1), \tag{2}$$

was made the subject of important recent investigations by W. Goncharoff[1] and J. M. Whittaker.[2] Goncharoff calls (1) a generalized Abel series; it reduces to the Taylor expansion of $f(x)$ about the origin if $\eta_0 = \eta_1 = \ldots = 0$. A discussion of the convergence of (1) for various types of sequences $\{\eta_n\}$ will lead to the problem of finding suitable estimates of $|\, p_n(x)\,|$. This last problem depends essentially upon the type of sequence $\{\eta_n\}$ under consideration. Goncharoff's estimate

$$|p_n(x)| \leqq \frac{(|x-\eta_0|+|\eta_0-\eta_1|+\ldots+|\eta_{n-2}-\eta_{n-1}|)^n}{n!} \tag{3}$$

is valid for arbitrary values of the η's and of x and is very favorable if η_n converges rapidly to a limiting value for $n \longrightarrow \infty$. In fact it yields readily the following theorem of Goncharoff:

If $f(x)$ is regular at the origin and $\eta_n \longrightarrow 0$ such that $\sum_1^\infty |\eta_{n-1}-\eta_n|$ converges, then the expansion (1) *is valid in a certain neighborhood of the origin. In particular, if for the same assumptions on the η's and $f(x)$ we also have*

$$f^{(n)}(\eta_n) = 0 \qquad (n = 0, 1, 2, \ldots), \tag{4}$$

then necessarily $f(x) \equiv 0$.

Whittaker investigated the expansion (1) in the case when every $\eta_n = +1$ or -1.* He finds for this case the estimate, valid if $|x| \geqq 1$,

$$|p_n(x)| \leqq {}^1/_2 e^2 (1+|x|)^n \quad (\eta_n = \pm 1), \tag{5}$$

which is better adapted to this particular situation ($\eta_n = \pm 1$) than Goncharoff's general estimate (3). By means of (5) Whittaker established the following theorem:**

Let $f(x)$ be an entire function of order < 1 or of order 1 and of type γ with

$$\gamma < \frac{1}{2}. \tag{6}$$

Let $\{\eta_n\}$ be a sequence with $\eta_n = +1$ or -1. Then the expansion (1) *is valid in the circle*

$$|x| < \frac{1}{\gamma} - 1. \tag{7}$$

In particular: *If besides our previous assumptions we also have* (4), *then $f(x) \equiv 0$.*

Whittaker notices the function $\sin \frac{\pi}{4}(x+1)$ of order 1 and type $\gamma = \pi/4 > 1/2$ which satisfies (4), for $\eta_n = (-1)^{n+1}$, without vanishing identically and conjectures that its type $\frac{\pi}{4}$ is the best constant on the right side of the inequality (6). A verification of this conjecture as well as an extension of Whittaker's results is contained in the following

THEOREM. *Let $f(x)$ be an entire function of order < 1 or of order 1 and of type γ with*

$$\gamma < \frac{\pi}{4} \tag{8}$$

Let $\{\eta_n\}$ be a sequence of real numbers with $-1 \leqq \eta_n \leqq +1$. Then the expansion (1) *is valid throughout the complex plane, absolutely and uniformly in every finite region.* In particular: *If a function $f(x)$ of the kind discussed presently is such that every one of its derivatives $f(x)$, $f'(x)$, $f''(x)$, . . vanishes somewhere between -1 and $+1$, then $f(x) \equiv 0$.*

This is established by means of the following estimate

$$|p_n(x)| < \left(\frac{4}{\pi}\right)^{n+1} 2\cosh\left(\frac{\pi}{4}|x|\right) + k\sqrt{2}\,e^{|x|}, \qquad \left(k = \frac{1}{\pi}\int_{|t|=1} \frac{|dt|}{|e^t + e^{-3t}|}\right), \tag{9}$$

which is valid provided $-1 \leqq \eta_\nu \leqq +1$ ($\nu = 0, 1, \ldots, n-1$).

This estimate is of the lowest possible order for large n. The estimate (5) yielded but the finite region (7) for the validity of (1), because the right side of (5) is of large order in n ($\longrightarrow \infty$) if $|x|$ is large. For a fixed value of n and $|x| \longrightarrow \infty$, (5) is obviously much superior to (9), but this is of no consequence in the present investigation.

An important step in the derivation of (9) is the solution of the following extremal problem:

Let $P_n(x)$ be a real polynomial of degree $n(>0)$ of the form $P_n(x) = x^n/n! + \ldots$. Let $P_n(x), P_n'(x), \ldots, P_n^{(n-1)}(x)$, all have at least one zero within $-1 \leqq x \leqq 1$. What is the largest value of $|P_n(0)|$?

This extremal problem is solved by the polynomial

$$P_n(x) = \int_{-1}^{x} dx_1 \int_{+1}^{x_1} dx_2 \ldots \int_{(-1)^n}^{x_{n-1}} dx_n.$$

* Whittaker actually considers the case $\eta_n = 0$ or 1 which is reduced to the case $\eta_n = +1$ or -1 by an obvious linear substitution.

** A weaker form of this theorem, the number $1/2$ on the right side of (6) being replaced by $1/2e$ is a particular case of a general theorem proved by Goncharoff and S. Takenaka.[3]

[1] W. Goncharoff, "Recherches sur les dérivées successives des fonctions analytiques, Généralisation de la série d'Abel," *Annales Sci. l'École Normale*, **47**, 1–78 (1930).

[2] J. M. Whittaker, "On Lidstone's Series and Two-Point Expansions of Analytic Functions," *Proc. Lond. Math. Soc.*, second series, **36**, 451–469 (1933–34).

[3] S. Takenaka, "On the Expansion of Analytic Functions in Series of Analytic Functions and Its Application to the Study of the Distribution of Zero Points of the Derivatives of Analytic Functions," *Proc. Physico. Math. Soc. Japan*, **13**, 111–132 (1931).

COLOR CHANGES IN CANCROID CRABS OF BERMUDA[1]

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THE BERMUDA BIOLOGICAL STATION FOR RESEARCH, INC.

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Color changes are limited among the invertebrates mainly to the cephalopods and to the crustaceans. They occur to some extent in leeches and in pteropods. The crustaceans, however, have been studied more thoroughly than any other group of invertebrates, and of the crustaceans the decapods have received by far the greatest amount of attention. Our knowledge of color physiology of crustaceans has progressed through the work on macruran decapods. The brachyurans have been investigated on only several occasions and it appears that they show a limited color display. This paper is concerned with a report of alterations in the distribution of several pigments in two brachyuran subgenera. These were identified by Dr. F. A. Chace as Portunus (Achelous) ordwayi (Stimpson) which I shall call the red crab, and Portunus (Portunus) anceps (Saussuri)—the white crab. Both species live on sandy bottoms and were obtained by dredging in Castle Harbor. They were not particularly abundant and in all about thirty specimens were collected during June and the early part of July.

A. *The Pigments of Portunid Crabs.*—The pigmentary system of the white crab consists of white, black and yellow pigments. These pigments are contained in separate chromatophores which are more numerous on the legs than elsewhere on the body. The white pigment bodies, without question the most numerous of the three, are distributed more or less uniformly over the dorsal surfaces of the appendages. In the central portion of the leg segments, especially the merus, the white pigment is absent or scarce, and in this region a cluster of black pigment bodies is situated. The yellow pigment, unlike the black, is uniformly distributed. Viewing as a whole the arrangement of these pigments on the dorsal surface of a leg, one sees a decided aggregation of all three pigments at its posterior margins. The chromatophores are most distinct in the joints between the leg segments, especially between the merus and the ischium, and between the basis and the coxa. In addition to the pigments already mentioned, a blue color was observed on the legs. This blue pigment did not appear to be confined to a special chromatophore but occurred as a haze in the vicinity of the black pigment.

In the red crabs there are red, white, yellow and blackish-brown pigments, which are supplemented by a golden-green iridescence on certain regions of the appendages. A blue coloration is also noticeable in the proximity of the red and dark pigments. Included among the taxonomic qualities which separate these species is the contrast in the color of the

bristles or hairs on the appendages. In *P. ordwayi* they are deep red, thus constituting a conspicuous element in its obvious pink coloration, while in *P. anceps* they are colorless. Another difference between the two species is the ability of the carapace of the red crab to assume different hues while that of the white crab shows little if any changes evident to the naked eye. In order of abundance the blackish-brown pigment dominates the red. There is a variation among different individuals, however, for specimens have been observed in which the most numerous chromatophores were red, whereas there were few dark bodies. The white pigment was more abundant than the yellow. These observations pertain only to the pigments of the legs, and are at best only rough approximations.

Finally, the pigments of these crabs like those of other crustaceans are contained in a chromatophore system. Chromatophores containing different pigments have never been observed to intermingle or anastomose with one another, but chromatophores containing the same pigment form anastomosing networks with each other. Such a condition is predominant for

TABLE 1

RESPONSES OF THE PIGMENTS OF WHITE CRABS TO DIFFERENT BACKGROUNDS

BACKGROUND	WHITE	BLACK	YELLOW
White	D	I	C
Black	C	D	D
Blue	C	D	C
Red	C	D	D
Yellow	C	D	C
Green	D	D	C
Darkness	C	D	D

the white, red and black pigments but it could not be decided whether this is also true for the yellow. Not all the chromatophores form networks, however, for many discrete chromatophores are found and in such bodies the following observations were made.

B. *Effect of Variously Colored Backgrounds.*—The changes in the distribution of the pigments in animals placed on various illuminated backgrounds are given in table 1 for the white crabs, and in table 2 for the red crabs. The tables represent a summary of the conditions in which (1) the animals were placed on the backgrounds directly after they were caught, and (2) they were changed from a background to which they had become equilibrated to one of its complementary color. The state of pigment distribution was recorded after periods of time varying from one hour to forty-eight hours, most of the observations being made during the daytime. Unfortunately the responses of the pigments were not consistent throughout and the observations recorded in the tables are true only of the majority of cases.

It would have been more satisfactory could these observations have been made on a larger scale for the variation in the response of the pigments was great, greater on some backgrounds than on others, and noticeable more in the red crabs than in the white. The yellow pigment especially showed great differences in reaction although part of the variation may be due to their indistinctness which would allow for a greater experimental error. Nevertheless, the data indicate that the chromatophores of adult brachyurans have not altogether lost the property of pigment alterations, although such changes are effective in altering only to a negligible degree the color of the animal as a whole.

C. *Effect of Blinding.*—Blinding experiments were carried out on the white crabs because they were more abundant. When animals adapted to darkness or to a black background, where in both situations the white pigment was concentrated and the black dispersed, were blinded and left in

TABLE 2

RESPONSES OF THE PIGMENTS OF RED CRABS TO DIFFERENT BACKGROUNDS

BACKGROUND	WHITE	BLACK	RED	YELLOW
White	D	C	C	C
Black	C	D	D	D
Blue	C	D	D	C
Red	C	D	D	D
Yellow	I	C	C	D
Green	D	D	I	C
Darkness	C	D	D	C

Symbols

D—dispersed
I—intermediate
C—concentrated

the light a wholly unexpected result was obtained. Within two hours, the black pigment became concentrated and the white dispersed. If the dark-room animals were, after blinding, returned to the dark room the same state in the condition of the black pigment was attained. Moreover, when animals equilibrated to a green background, where both the white and the black pigments were in a dispersed state, were blinded and left subsequently in the light, the same responses were evoked: the black became concentrated and the white remained dispersed. Since no situation was found among the diverse backgrounds where the black and the white pigments were simultaneously concentrated the effect of blinding on this condition could not be determined.

That the black pigment becomes concentrated following loss of the eyes is an observation which stands in sharp contrast to the universal effect of blinding on melanophores. In practically every crustacean with the ex-

ception of Uca (Megasur, 1912; Carlson, 1935) ablation of the eye-stalks is followed by an expansion of the main color cells (cf. p. 448, Parker, Brown and Odiorne, 1935). In amphibians, reptiles and fishes, the universal effect of optic enucleation is a dispersion of melanophore pigment. A complete and irreversible concentration of black pigment, however, is brought about in amphibians, some reptiles and lower fishes by the removal of the pituitary gland. The concentration of black pigment after loss of the eyes in Portunid crabs has then a striking parallel in the effect of hypophysectomy on vertebrate melanophores. Viewed in the light of recent advances in color physiology of crustaceans, this unorthodox situation in brachyuran crabs becomes understandable. The eye-stalks of numerous decapods contain a gland or glands (Perkins, 1928; Kropp and Perkins, 1933; Hanström, 1935) which by their secretions bring about a concentration of body pigment cells. When these glands are extirpated as by blinding an expansion of the chromatophores results. It is entirely feasible that among other decapods such glands may maintain a dispersion of pigment and their absence (as in blinded Portunus) may lead to a concentration of black pigment masses. It may also be possible that such glands are lacking in the eye-stalks of Portunus as they are in Gebia and Hippa (Hanström, 1935).

Unfortunately, the experiments were terminated at this point. It was indeed gratifying to learn that Carlson (1935), who has just published his results with Uca, carried this aspect of the problem to its logical completion. Uca becomes pale following the removal of its eyes. Injection of extracts of the eye-stalks into blinded animals restores the normal dark coloration. The effect of the eye-stalk hormone on the black chromatophores of blinded crabs and the effect of the melanophore principle of the pituitary on melanophores of certain hypophysectomized vertebrates are identical, and the similarity of the color responses of these two groups is greatly enhanced. The action of the eye-stalk hormone, however, is more complicated than that of the hypophysis for Carlson has shown that extracts of Uca eye-stalks produce a contraction of the chromatophores of Palaemonetes, and that Palaemonetes extracts darken blinded Uca. *Palaemonetes vulgaris*, however, has no black pigment cells (Brown, 1935) and consequently it would appear that the factor determining the action of the eye-stalk hormone resides with the particular type of pigment acted on. This opinion seems temporarily to be more satisfactory than an alternative one which would necessitate the presence of a hormone in the eye-stalks of Palaemonetes for a black integumentary pigment which it does not contain

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NOTE ON THE EFFECT OF LIGHT ON THE BIOELECTRIC POTENTIALS IN THE AVENA COLEOPTILE

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The effect of radiant energy on bioelectric potentials in plants has been little investigated chiefly because of the complexity of interpretation of the results. All previous work of this type has been confined to bioelectric responses of green organs.[1,2,3,4] Thus the interpretation becomes difficult as photosynthesis itself presumably causes changes in potentials which are superimposed upon the normal bioelectric potentials. The present preliminary note will briefly describe bioelectric responses of the coleoptile of the etiolated seedling of the oat, an organ totally lacking in chlorophyll. In this case it was hoped to obtain the more or less direct effect of light on the potentials of a plant organ.

The experiments were carried out with etiolated seedlings of a pure line of *Avena sativa* (Sieges Hafer) grown at 24°C. and a relative humidity of 90. When the seedlings had reached a length of 30–40 mm., one of them was transferred without injury and totally intact to the experimental chamber. Only weak red light was employed for the observations. The chamber contained two partitions, one for the experimental plant and one for the control plant. Each partition contained a special holder for the seed whose roots were suspended in a vial of water. The coleoptile extended upward through four glass loops placed equi-distant from one another ranging from the coleoptilar node to the tip. Each glass loop held a tap water meniscus which in turn made water contact to side-arms of cups of water in which Zn amalgam-saturated $ZnSO_4$ half-cells were placed. Thus four fixed electrical contracts were made to each coleoptile so that the potential difference between any two of them could be measured. They are illustrated as *A*, *B*, *C* and *D* in the figure. These contacts are essentially the same as those

used by Glass[4] in his study of the effect of light on the bioelectric potentials in Elodea leaves. No $ZnSO_4$ reached the plant through the side-arms within the time of any one experiment. All contacts were isoelectric to one another. The four contacts made connection by well insulated wires to mercury-in-paraffin switches outside the light-proof chambers. The measurements were made with a single string electrometer of the *Wulf* type.[5] The instrument was constructed in the physics shops of this insti-

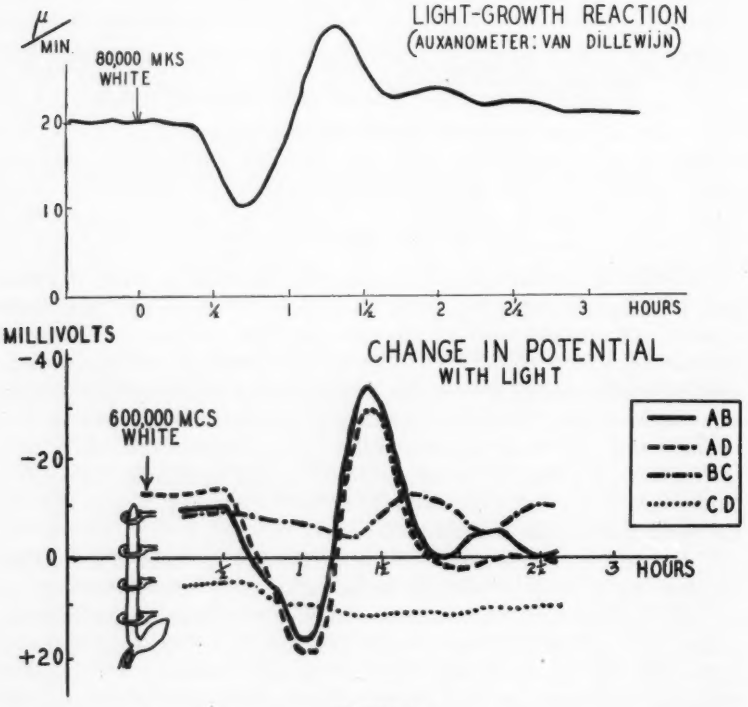

FIGURE 1

tution. Its period was less than a second with a sensitivity of one millimeter scale division per millivolt. The sensitivity remained constant over periods of weeks if the instrument was kept at constant temperature. Because of its simplicity and rapidity of operation, this instrument has proved itself extremely useful for work of this type.

The light source for the stimuli was a 100-watt Mazda incandescent lamp 40 cm. above the tip of the seedling, being horizontally directed to the plant from three sides by means of three mirror strips placed 120°

from one another and 45° to the horizontal. The heat radiation was filtered out by the interposition of a layer of one centimeter of running water between light source and chamber, so that the temperature of both partitions of the experimental chamber remained constant within 0.1°C. while either partition was being illuminated.

Preliminary measurements of plant electrical potentials obtained by contacts moved up and down the plant by means of micro-manipulators and under red or orange light, and at constant temperature, were extremely variable and it was impossible to obtain constancy over any considerable length of time after the most careful manipulation. Only those plants with four *fixed* contracts and in *complete darkness* gave the constancy necessary before the electrical responses to any external stimulus could be determined. With four such contacts, the potential differences (P. D.) between *A* and *B*, *B* and *C*, *C* and *D* and their sum *A* and *D*, were determined by manipulation of the mercury switches. These values were plotted as millivolts. The figure shows the results. Approximately 90 to 120 minutes after setting up the plant elapsed before constancy of the P. D.'s obtained. The changes previous to this usually consisted of a slow fall followed by a slow rise in negativity of the coleoptile tip with respect to the more basal contacts. With respect to time relations and shape of the curve, this reaction corresponds closely with the growth reactions obtained after setting up a similar plant in a Koningsberger auxanometer[6] as was seen by examination of many auxanometer records.

After constancy was obtained, the light was turned on one plant in one partition while the other plant set up in exactly the same way was kept in complete darkness in the other partition. Measurements were made in a few seconds every five or ten minutes. After the reaction of the illuminated plant was recorded and relative constancy again reached, the other plant was illuminated for its reaction, and so on. Usually two such reactions were recorded in one day's experimentation. The time of illumination was varied, but it was found that with the times most studied, namely 5, 10 and 30 minutes, the curves were similar. Since it was very difficult to obtain a set-up wherein the potentials were constant before illumination, many experiments were necessarily performed before a few good ones were obtained. This condition was somewhat offset by dipping the glass loops in gelatin or agar before mounting the contacts on the plants. Presumably "handling reactions" are caused by the plant rubbing against the solid glass loops (cf. Pfeffer[7]).

Of the experiments performed, seven wherein the period of illumination was thirty minutes gave similar curves. These are typified by the curve given in the figure.

It is noticed that the electrical response comes after the light is turned off. (One hundred watts at 40 cm. for 10 min. = approximately 600,000

meter-candle-seconds, assuming 167 for the mean horizontal candle-power of the 100-watt gas-filled incandescent lamp.) The tip of the coleoptile is normally electronegative to the base, in contradiction to Ramshorn,[8] who used manipulators to move the contacts up and down the plants, making observations by red light. After illumination, this negativity at first decreases, then increases to a maximum after which it falls again to the original level, usually fluctuating somewhat thereafter. The largest reaction obtains in the sub-apical region, *AB*, with smaller delayed, reactions in the more basal regions. The algebraic sum of the P. D.'s between *A* and *B*, *B* and *C* and *C* and *D* is always equal to that between *A* and *D*. The time relations, the magnitude of the effect (up to 90 millivolts) and the shape of the curve all strongly suggest a relationship to the light-growth reactions of the Avena coleoptile.[9,10] A curve from van Dillewijn (p. 438)[9] for the light-growth reaction is given in the figure for comparison of shape of the curves and time relations. Whether the electrical reaction is a cause, an effect or a parallel phenomenon possibly associated with growth reactions to light remains to be shown. A closer analysis of the factors involved is now in progress.

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